A MODIFIED ALGORITHM FOR FIXED CHARGE BI-CRITERION TRANSPORTATION PROBLEM WITH RESTRICTED FLOW

Thesis submitted in partial fulfillment of the requirement for

The award of the degree of

Masters of Science

In

Mathematics and Computing

Submitted by

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JULY 2011

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CERTIFICATE -

I hereby certify that the work which is being presented in the thesis entitled "**A modified algorithm for fixed charge bi-criterion transportation problem with restricted flow**" in partial fulfillment of the requirements for the award of degree of Master of Science, School of Mathematics and Computer Applications, Thapar University, Patiala is an authentic record of my own work carried out under the supervision of Dr. Mahesh Kumar Sharma and Mr. Vikas Sharma.

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university

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ACKNOWLEDGEMENT

The completion of this thesis has involved a lot of people to whom I would like to express my sincere thanks and gratitude for their help.

For most, I would like to pass my appreciation and gratitude to my honorable supervisors, ***Dr. Mahesh Kumar Sharma****, Assistant Professor and* ***Mr. Vikas Sharma,*** *lecturer, School of Mathematics and Computer Applications, Thapar University, Patiala. For their constructive suggestions, detailed corrections, support and encouragement in accomplishing this work. Moreover, for mentoring me when I needed the most. I am fortunate that I got an opportunity to work under their supervision.*

I express my regards and gratitude to ***Dr. S.S. Bhatia****, Head of Department, School of Mathematics and Computer Applications, Thapar University, Patiala, for providing keen interest, unfailing support, inspiration and necessary research facilities in the school.*

I am thankful to ***Mr. Gourav Gupta****, School of Mathematics and Computer Applications, Thapar University, Patiala. For his help when it is need to complete thesis.*

I would like to thank my beloved parents for their unconditional support and deep trust in me, without whom my project would have been a mere dream rather than a reality.

I would also thank all the academic and administrative staff of School of Mathematics and Computer Applications, Thapar University, Patiala.

Finally, I am also thankful to my friends who also contributed a lot in accomplishing this piece of work.

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ABSTRACT

The fixed- charge transportation problem (FCTP) is an extension of the classical transportation problem in which a fixed cost is incurred, independent of the amount transported, along with a variable cost that is proportional to the amount shipped. The introduction of fixed costs in addition to variable costs results in the objective function being a step function.

The fixed charge bi-criterion transportation problem with restricted flow (FCTP with restricted flow) which is an extension of the fixed charge bi-criterion transportation problem has been studied in the present work. In the fixed charge bi-criterion transportation problem a fixed cost called set up cost is incurred for every origin. In the bi-criterion transportation problem the cost of transportation is directly proportional to the number of units transported, but on account of quantity discounts, price breaks etc.

The present thesis contains three chapters. In Chapter one is introductory in nature.In chapter second, the algorithm given by Thirwani *et. al*. (1997) has been modified and more number of cost-time trade off pairs are (FCTP with restricted flow) obtained. In chapter three, (FCTP with restricted flow) has been modified and solved by using the algorithm developed in chapter second.

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CHAPTER 1

"INTRODUCTION"

1. INTRODUCTION

The transportation problem is one of many well-structured problems in operation research that has been extensively studied in literature. The transportation is amongst the most important special linear programming problem. Generally the linear programming problems are similar in the sense that they require a linear function to be optimized where the variable satisfy certain linear constraints. These problems can invariably be solved using simplex method but a number of alternative methods have been constructedfor problems like transportation problem. The commonly used method to solve classical transportation problem is MODI method or (U. V method).The classical transportation problem arises when we must determine an optimal schedule of shipment that:

(a) Originate at sources(supply depots) where fixed stock piles of a commodity are available.
(b) Are sent directly to their fixed destination (demand depots) where various fixed amount are required.
(c) Exhaust the stock piles and fulfill the demand. Hence total demand equals to total supply and finally the cost must satisfy a linear objective function. That is, the cost of each shipment is proportional to the amount shipped and the total cost is the sum of the individual costs.

Mathematically this problem can be formulated as:

Minimize $$z = \sum_i \sum_j c_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \ , \quad a_i > 0, i = 1,2,...,m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \ , \quad b_j > 0, j = 1,2,...,n$$

$$x_{ij} \geq 0 \quad for\ all\ \ i\ \ and\ \ j$$

a_i is the quantity of the product available at origin i

b_j is the quantity of the product required at destination j

$c_{ij}$is the cost of shipping one unit from origin i to destination j

x_{ij}amount shipped fromi^{th} source toj^{th}destination.

It is assumed that total availability is equal to total demand, i.e. $\left(\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j\right)$

but in certain real life situation ,the total availability may not be equals to the total requirement , i.e. $\left(\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j\right)$ for example, if supply is greater than demand $\left(\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j\right)$ then a fictitious destination may be used to create the desired equality. If the demand exceeds supply $\left(\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j\right)$, then a fictitious source may be introduced.

1.2 TIME MINIMIZING TRANSPORTATION PROBLEM

Sometimes there may exists emergency situations such as those requiring police services, fire services, ambulance services, etc., when the time of transportation is of greater importance than cost of transportation. Some methods for minimizing the time of transportation have been established. In such situations rather than minimizing the cost, the objective is to minimize the maximum time to transport all supply to destinations satisfying certain conditions in respect of availabilities at sources and requirements at the destinations.Thus, a time minimizing transportation problem can be formulated mathematically as:

Minimize $$\left[\max_{(i,j)} t_{ij} \,/\, x_{ij} > 0\right]$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \,,\ (i = 1,2,...,m),$$

$$\sum_{i=1}^{m} x_{ij} = b_j \,,\ (j = 1,2,...,n).$$

$x_{ij} \geq 0 \quad for\ all\ i\ and\ j$

Here $t_{ij}$is the time of transporting goods from the i^{th} origin, where the availability is a_i to the j^{th}destination, where the requirement is b_j. For any given feasible solution, $X = [x_{ij}]$ satisfying the above constraints, the time of transportation is the maximum of t_{ij}*'s*among the cells in which there are positive allocations, i.e., corresponding to the solution $X = [x_{ij}]$, the time of transportation is

$$Z = \left[\max_{(i,j)} t_{ij} \,/\, x_{ij} > 0 \right]$$

The aim is to minimize this time of transportation.

1.3 FIXED CHARGE TRANSPORTATION PROBLEM

The fixed charge transportation problem (FCTP) is an extension of classical transportation problem in which fixed cost is incurred for every origin. The fixed charge transportation problem was originally formulated by Hirsch and Dantzig. Many distribution problems in practice can only be modeled as FCTPs. For example, rails, roads and trucks have invariable used freight rates which consist of a fixed cost and a variable cost. The fixed cost may represent the cost of renting a vehicle, landing fees at airport, set up costs for machines in manufacturing environment etc.

The problem can be formulated mathematically as:

Minimize $$\{\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij} + F_i\}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \;, \quad a_i > 0, i = 1,2,...,m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \;, \; b_j > 0, j = 1,2,...,n$$

$$x_{ij} \geq 0 \quad for\ all\ i\ and\ j$$

a_i is the quantity of the product available at origin i

b_j is the quantity of the product required at destination j

$c_{ij}$is the cost of shipping one unit from origin i to destination j

x_{ij}amount shipped fromi^{th} source toj^{th}destination.

F_i the fixed cost associated with origin i.

1.4 MULTI-CRITERIA OPTIMIZATION

Multi-criteria optimization (or multi-objective programming),also known as multi-criteria or multi-attribute optimization, is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints.

Multi-criteria optimization problems can be found in various fields: product and process design, finance, aircraft design, the oil and gas industry, automobile design, or wherever optimal decisions need to be taken in the presence of trade-offs between two or more conflicting objectives

For nontrivial multi-criteria optimization problems, one cannot identify a single solution that simultaneously minimizes each objective to its fullest. While searching for solutions, one reaches points such that, when attempting to improve an objective further, other objectives suffer as a result. A solution is called non-dominated if it cannot be eliminated from consideration because there is at least another solution which improves an objective without worsening another one. Finding such non-dominated solutions, and quantifying the trade-offs in satisfying the different objectives, is the goal when setting up and solving a multi-criteria optimization problem.

In general a multi-criteria programming problem can be formulated as:

Optimize $f(x) = (f_1(x), f_2(x), ..., f_k(x))$

Subject to

$$g_j(x) \leq = \geq b_j, \; j = 1, 2, ..., m$$

$$x \geq 0$$

$$X = (x_1, x_2, \dots, x_n)^T$$

Where, $f(x)$ is the objective function to optimize. $f_1(x), f_2(x),...., f_k(x)$ are k number of distinct objective function subject to m constraints. X is a vector consists of decision variables $x_1, x_2, \ldots, x_n$.

Example: We want to buy a new car and have identified four models welike: a VW Golf, an Opel Astra, a Ford Focus and a Toyota Corolla. Thedecision will be made according to price, petrol consumption, and power.We prefer a cheap and powerful car with low petrol consumption. In thiscase, we face a decision problem with four alternatives and three criteria. Thecharacteristics of the four cars are shown in Table (data are invented).How do we decide, which of the four cars is the "best" alternative, whenthe most powerful car is also the one with the highest petrol consumption, sothat we cannot buy a car that is cheap as well as powerful and fuel efficient.However, we observe that with any one of the three criteria alone thechoiceis easy.

Criteria and alternatives in Example 1

Criteria	Alternatives: VW	Opel	Ford	Toyota
Price(1,000Euros)	16.2	14.9	14.0	15.2
Consumption (1/100 km)	7.2	7.0	7.5	8.2
Power (kW)	66.0	62.0	55.0	71.0

1.5 FIXED CHARGE BI-CRITERIA TRANSPORTATION PROBLEM

From a practical point of view, the cost minimizing transportation problem and time minimizing transportation problem cannot be viewed as two independent problems. If one is interested in obtaining a solution which minimizes cost and time simultaneously is called bi-criteria transportation problem.

The fixed charge bi-criteria transportation problem is an extension of the bi-criteria transportation problem which is more complex to solve. In this type of problem a fixed charge called a setup charge is incurred for every origin. In the classical transportation problem the cost of transportation is directly proportional to the number of units transported, but on account of quantity discounts, price breaks etc. the transportation may not be linear. Thus in real world situations, when a commodity is transported, a fixed cost is incurred in the objective function. The fixed cost may represent the cost of renting a vehicle, landing fees in an airport, setup costs for machines in a manufacturing environment, a new facility costs money to be constructed etc. It also costs money to operate.

Therefore, a fixed charge bi-criteria transportation problem (FCTP) formulated as:

$$\text{Minimizing} \quad \left\{ \sum_{i\in I} \sum_{j\in J} c_{ij}x_{ij} + \sum_{i\in I} F_i \,,\ \max_{\substack{i\in I \\ j\in J}} [t_{ij} \,|x_{ij} > 0] \right\}$$

Subject to

$$\sum_{j\in J} x_{ij} = a_i, \quad i \in I$$

$$\sum_{i\in I} x_{ij} = b_j, \quad j \in J$$

$$x_{\,ij} \geq 0 \quad i \in I, \qquad j \in J$$

Where $I = \{1, 2, . . .,m\}$ is the set of origins.

$J = \{1, 2, . . .,m\}$ is the set of destinations.

$x_{i\,j}$ = the amount transported from the i^{th} origin to the j^{th} destination,

c_{ij}= the variable cost per unit amount transported from i^{th} origin to the j^{th} destination,

t_{ij}= the time of transportation of the product from i^{th} origin to the j^{th} destination which is independent of the amount of commodity transported, so long asx_{ij}> 0,

a_{i} = maximum capacity at origin i,

b_j= the demand at destination j,

F_i = the fixed cost associated with origin i.

LITERATURE SURVEY

There are different types of transportation problems and the simplest of them is now standard in the literature was first presented by Hitchcock(1941). It usually aims to minimize the total transportation cost. Other objectives that can be set are a minimization of the total delivery time, a maximization of profits, etc. from the investigation; the entire existing objectives in single objective transportation model are represented by quantitative information. This may cause the negligence of some crucial points which cannot be described by quantitative data.

Laterindependently,by Koopman(1947).Koopman began to spearhead research on the potentialities of linear programs for the study of the problems in economics. His historic paper "optimum utilization of the transportation potations systems" was based on his war time experience. Because of this and the work done earlier by Hitchcock, the classical case is often referred as the Hitchcock-Koopman's transportation problem. Kantorovich (1942) publishes the paper on a continuous version of the problem and later with Gavurin, an applied study of the capacitated transportation problem (Kantorovich and Gavurin 1949).

The time minimizing transportation problem has been studied by Hammer (1969), Garfinkel and Rao (1971) and Szwarc (1971). Hammer (1969) and Szwarc(1971) used labeling techniques to solve the problem(I). Garfunkel and Rao(1971) solved the problem by introducing a sufficiently large cost M on certain routes. Sometimes there may exist emergency situations such as those requiring police services, fire services, ambulance services, etc., when the time of transportation is of greater importance than cost of transportation. Some methods for minimizing the time of transportation have been established.Several methods for minimizing the time of transportation are also developed. Then Bhatia *et. al.* (1975) developed a technique for minimizing time in a transportation problem. The procedure involved finite number of iterations and is based on moving from one basic feasible solution to another till the last solution is arrived at. The algorithm given by them consists of determination of an initial basic feasible solution which can be found by the methods applicable in the case of the common cost minimizing transportation problem and finding an adjacent better basic feasible solution, the procedure is repeated until no better adjacent basic feasible solution can be found. This repeated procedure deals with the determination of a cell not in the basis which, when introduced, will either reduce the time of transportation or reduce the allocation in at least one of the cells $\in$ Q, where Q is the set of cell with positive allocations and corresponding time equal to the time of transportation.

The transportation problem with two-objectives known as the bi-criterion transportation problem has been studied by many research workers. In this type of problem there are two objectives - one primary and the other secondary. The primary objective is to minimize the total cost of transportation problem and the secondary objective is to minimize the duration of transportation.

In a classical transportation problem the cost of transportation is directly proportional to the number of units of the commodity transported. But in real world situations when a commodity is transported, a fixed cost is incurred in objective function. Fixed charge transportation problem has been investigated by many research workers in which a fixed charge is associated with each route that can be opened, in addition to the variable transportation cost proportional to the amount of goods shipped. The fixed charge problem was first formulated by Hirsch and Dantzig in (1954). In (1961)Balinskishowed the fixed charge problem to the special case of the fixedcharge problem and presented as approximate solution. In (1968)Murty devised an exact solution based on searching among the adjacent extreme points of the transportation problem; however, he presented only one sample problem, solved by hand. He pointed out that the method was the most useful for the case in which the fixed charge quite small compared to transportation cost. Paul Gray (1968) uses an alternate approach to Murty's, namely a search among the extreme points according to their associated fixed charges. To improve computability, the special structure of the transportation problem is exploited extensively. David R. Denzler described an approximate solution method for solving the fixed charge problem. This heuristic approach is applied to the set of test problems to explore the margin of error. The result indicates that the proposed fixed charge simplex algorithm is capable of finding optimal or near optimal solutions to moderates the fixed charge problems. In the absence of an exact method, this heuristic should prove useful in solving this fundamental nonlinear programming problem. Also Basu*et. al.*(1994) developed an algorithm for the optimum time-cost trade-off in a fixed charge linear transportation problem giving same priority to cost and time.Puri and Swarup (1974) established a method of solving the fixed charge problem by breaking it into two problems, one a simple linear programming problem and other a zero-one problem with no constraints of usual type.Bhatia *et. al.*(1976) provided the time-cost trade off pair in a transportation problem. Barr et.al. (1981) presented a branch and bound algorithm for solving large scale fixed charge transportation problem where all cells do not exist. The algorithm exploits the absence of full problem density in several ways, thus yielding a procedure which is especially applicable to solving real world problems which are normally quite sparse. Cooper and Drebes(1967)provided two heuristic methods for moderate sized problems and the results indicate that the heuristic methods produce optimal solutions in well over 90 percent of the several hundred problems investigated and very close to optimal (a few percent) in the remaining cases. Sandrock (1988) presented a low technology algorithm for the solution of small, fixed charge problems. Prasad *et. al.* (1993) considered that the unit costs are piecewise linear non-increasing functions of time and it is assumed that all the transportations may take place concurrently. As such, the trade-offs they seek are between the total cost and the bottleneck time for completing all the transportations to all the demand points. They showed that a parametric method involving a finite sequence of parametric transportation problems revealed all the time-cost trade off solutions of the generalized trade-off problem. Also, a direct method is outlined for the case involving a finite set of discrete alternatives of unit cost-time pairs for each pair of supply-demand points.Balinski (1961) has given an approximate method of solving the fixed charge problem.Gottileb and Paulmann (1998) presented two genetic algorithms for FCTP. Both algorithms incorporate knowledge about the properties of optimal solutions. The algorithms mainly differ in the technique used to deal with the inherent constraints of FCTP they compared both genetic algorithms on randomly generated instances.

Recently, Basue*et. al. (1994)* developed a technique for solving the fixed charge bi-criterion transportation problem. Arora*et. al.* (1996) developed a fixed charge bi-criterion transportation problem wherein there is a restriction on the total flow is studied. An algorithm to find the efficient cost- time trade off pairs in a fixed charge bi-criterion transportation problem with restricted flow is presented. Gottileb and Paulmann (1998) presented two genetic algorithms for FCTP. Both algorithms incorporate knowledge about the properties of optimal solutions. The algorithms mainly differ in the technique used to deal with the inherent constraints of FCTP; they compared both genetic algorithms on randomly generated instances.

PRESENT WORK

The algorithm given by Thirwan*et. al.* (1997) has been improved to get more cost-time trade off pairs for (FCTP with restricted flow). In chapter 3, (FCTP with restricted flow) has been modified the fixed charge and solved by using the algorithm developed in chapter 2.

CHAPTER- 2

“ MODIFIED ALGORITHM FOR FIXED CHARGE BI-CRITERIA TRANSPORTATION PROBLEM WITH RESTRICTED FLOW”

MODIFIED ALGORITHM FORFIXED CHARGE BI-CRITERIA TRANSPORTATION PROBLEM WITH RESTRICTED FLOW

2.1 INTRODUCTION

Bi-criteria transportation problem is an extension of single objective transportation problem. In this type of problem there are two objectives: one of minimizing the total cost and the second is to minimize the total time of transportation.

The fixed charge bi-criterion transportation problem is an extension of the bi-criterion transportation problem which is more complex to solve. In this type of problem a fixed cost called a setup cost is incurred for every origin. In the classical transportation problem the cost of transportation is directly proportional to the number of units transported, but on account of quantity discounts, price breaks etc. the transportation may not be linear. Thus in real world situations, when a commodity is transported, a fixed cost is incurred in the objective function. The fixed cost may represent the cost of renting a vehicle, landing fees in an airport, setup costs for machines in a manufacturing environment, a new facility costs money to be constructed etc. It also costs money to operate.

In this chapter an algorithm given by Thirwani *et. al.* (1997) for fixed charge transportation problem with restricted flow has been modified to get more cost-time trade off pairs

2.2 FORMULATION OF FIXED CHARGE BI-CRIERION TRANSPORTATION PROBLEM WITH RESTRICTED FLOW

If the total transportation flows to a known specified $P\ \left(< min\left(\sum_{i\in I} a_i\,,\ \sum_{j\in J} b_j\right)\right)$, this flow constraint breaks the transportation structure of the problem. The resulting fixed charge bi-criterion transportation problem with restricted flow is given by Thirwani *et. al.* (1997) as

$$\text{Minimize}\left\{\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij} + \sum_{i\in I} F_i\,,\ \max_{\substack{i\in I\\ j\in J}}[t_{ij}\,|x_{ij} > 0]\right\}$$

Subject to

$$\left.\begin{aligned} &\sum_{j\in J} x_{ij} \le a_i, \quad i \in I \\ &\sum_{i\in I} x_{ij} \le b_j, \quad j \in J \\ &\sum_{i\in I}\sum_{j\in J} x_{ij} = P\left(< min\left(\sum_{i\in I} a_i\,, \quad \sum_{j\in J} b_j\right)\right) \\ &x_{ij} \ge 0 \quad i \in I, \quad j \in J \end{aligned}\right\} \qquad \textbf{(2.1)}$$

x_{ij} = the amount transported from the i^{th} origin to the j^{th} destination,

c_{ij}= the variable cost per unit amount transported from i^{th} origin to the j^{th} destination,

t_{ij}= the time of transportation of the product from i^{th} origin to the j^{th} destination which is independent of the amount of commodity transported, so long asx_{ij}> 0,

a_i = maximum capacity at origin i,

b_j= the demand at destination j,

F_i = the fixed cost associated with origin i.

2.3 SOLUTION PROCEDURE:

For the solution of (FCBTP with restricted flow), the algorithm given by Thirwan*et. al.* (1997) has been modified by using reoptimizing procedure and theoretical development given byThirwan*et. al.* (1997)which are reported for ready reference.

(FCBTP) is converted into two problems (P1) and (P2) where

(P1): Minimize$\left[\sum_{i\in I}\sum_{J\in j} c_{ij}x_{ij} + \sum_{i\in I} F_i\right]$

Subject to (2.1)

and

(P2): Minimize $[\max_{\substack{i\in I\\ j\in J}}[t_{ij}\,|x_{ij}>0]]$

Subject to (2.1)

For formulation of $F_i\,(i=1,2,\ldots,m)$ it is assumed that $F_i\,(i=1,2,\ldots,m)$ has p number of steps so that

$$F_i=\sum_{i=1}^{p}\delta_{il}\,F_{il}\,,\quad i=1,2,\ldots,m$$

Where $\delta_{il}=1$ if $\sum_{j=1}^{n}x_{il}>A_{il},\quad i=1,2,\ldots,m;\ l=1,2,\ldots,p$

= 0 otherwise

Here $0=A_{i1}<A_{i2}<\ldots<A_{ip}$

$A_{i1},A_{i2},\ldots,A_{ip}\ (i=1,2,\ldots,m)$ are constants and F_{il} ($i=1,2,\ldots,m;\ l=1,2,\ldots,p$) are fixed costs.

The flow constraint in the problem (P1) ensure that a total amount $\left(\sum_{i\in I}a_i-P\right)$ of source reserves has to be kept at the various sources and a total amount $\left(\sum_{j\in J}b_j-P\right)$ of destination slacks to be retained at the various destinations to receive the source reserves and an extra source to fill up the destination slacks are introduce. Hence, the related fixed charge bi-criterion transportation problem (RFCBTP) associated with fixed charge bi-criterion transportation problem (FCBTP) is:

(RFCBTP) Minimize$\left\{\sum_{i\in I'}\sum_{j\in J'}c'_{ij}y_{ij}+\sum_{i\in I}F_i\,,\ \text{Max}[t'_{ij}\,|y_{ij}>0]\right\}$

Subject to

$$\left.\begin{aligned}&\sum_{j\in J'}y_{ij}=a'_i\,, && i\in I'\\ &\sum_{i\in I'}y_{ij}=b'_j\,, && j\in J'\\ &\sum_{i\in I'}\sum_{j\in J'}y_{ij}=P\\ &y_{ij}\geq 0,\ i\in I',\ j\in J'\end{aligned}\right\}\qquad (2.2)$$

$$where \;\; I^{'} = \{1, 2, \ldots, m+1\} = I \cup \{m+1\}$$

$$J^{'} = \{1, 2, \ldots, n+1\} = J \cup \{n+1\}$$

$$a_i^{'} = a_i, \; i \in I, \; a_{m+1}^{'} = \left(\sum_{j \in J} b_j - P \right)$$

$$b_j^{'} = b_j, \; j \in J, \; b_{n+1}^{'} = \left(\sum_{i \in I} a_i - P \right)$$

$$c_{ij}^{'} = c_{ij} (i, j) \in I \times J$$

$$t_{ij}^{'} = t_{ij} (i, j) \in I \times J$$

$$c_{i,n+1}^{'} = c_{m+1,j}^{'} = 0, \; i \in I, \; j \in J$$

$$t_{i,n+1}^{'} = t_{m+1,j}^{'} = 0, \; i \in I, \; j \in J$$

$$F_{m+1} = 0,$$

$$c_{m+1,n+1}^{'} = M, \qquad t_{m+1,n+1}^{'} > \underset{\substack{i \in I^{'} \\ j \in J^{'}}}{Max} [t_{ij} \mid y_{ij} > 0]$$

Where M is a large positive number.

(RFCBTP) is separated into two problems (RP1)and (RP2).

(RP1): Minimize $\left\{\sum_{i \in I'} \sum_{j \in J'} c'_{ij} y_{ij} + \sum_{i \in I} F_i\right\}$

Subject to (2.2)

(RP2): Minimize $T = \underset{\substack{i \in I' \\ j \in J'}}{Max} [t_{ij} | y_{ij} > 0]$

Subject to (2.2)

2.4 THEORETICAL DEVELOPMENTS

Thirwani *et. al.* (1997) developed some theory related to this problem and is given below.

Definition: A basic feasible solution $\{y_{ij}\}, \; i \in I', j \in J'$ to (RFCBTP)

is called a corner feasible solution (cfs) if $y_{m+1,n+1} = 0$.

Theorem 1:*every corner feasible solution of (RFCBTP) provides a basic feasible solution to (FCBTP) and conversely.*

Remarks 1: the value of the objective function of (RP1) at a corner feasible solution is equal to the value of the objective function of (P1) at its corresponding basic feasible solution.

Value of the objective function of (RP1) is

$$= \sum_{i\in I'} \sum_{j\in J'} c'_{ij} y_{ij} + \sum_{i\in I'} F_i$$

$$= \sum_{i\in I'} \sum_{j\in J'} c'_{ij} y_{ij} + \sum_{i\in I'} c'_{i,n+1} y_{i,n+1} + \sum_{i\in I'} F_i \ \ (because\ F_{m+1} = 0)$$

$$\sum_{i\in I} \sum_{j\in J} c'_{ij} y_{ij} + \sum_{j\in J} c'_{m+1,j} y_{m+1,j} + \sum_{i\in I'} c'_{i,n+1} y_{i,n+1} + \sum_{i\in I'} F_i$$

$$= \sum_{i\in I'} \sum_{j\in J'} c'_{ij} y_{ij} + \sum_{i\in I'} F_i \ \ (\because c'_{m+1,j} = c'_{i,n+1} = 0, y_{m+1,n+1} = 0$$

= value of the objective function of (P1).

Remarks 2: A non-corner feasible solution to (RFCBTP) cannot provides a feasible solution to (FCBTP).

Theorem 2: *An optimal solution to (RP1) has to be a corner feasible solution.*
Theorem 3:*There is a one to one correspondence between optimal solutions to* (P1) *and optima among the corner feasible solutions to(RP1).*

Conclusion: Optimizing (P1) is exactly equivalent to optimizing (RP1) which is a fixed charge transportation problem whose optimal solution is at an extreme point by Murty (1968)

ALGORITHM

Step 1: K is the number of iterations in the algorithm and let a set $S=\phi$.

Step2: Given the fixed charge bi-criterion transportation problem, separated into two problems (P1) and (P2).Let the flow be restricted to P. Introduce an additional row and an additional column with availability = $\sum_{i\in I} a_i - P$ and demand = $\sum_{j\in J} b_j - P$. Find its basic feasible solution $\{y_{ij}^k\}$ and let B be the corresponding basis.

Step 3: Calculate the fixed cost of the current basic feasible solution and denote this by F^k (current) where

$$F^k(current) = \sum_{i\in I} F_i$$

Step 4: Let $(Z_k)=(F^k)+(z_0{}^k)$ be the total cost corresponding to basic feasible solution $\{y_{ij}^k\}$ of P(1). Here, F^k is fixed cost and $z_0{}^k$ is a variable cost

Step 5: Max time correspond to $\{y_{ij}^k\}$ is

$T_k = \max\{t_{ij} \mid x_{ij} > 0, (i, j) \in B\}$

Step6: Find set $S = \{(Z_k, T_k)\} \cup S$

Step6(a): If $S=\phi$ then S becomes

$S = \{(z_k, T_k)\}$

and go to step 7.

Step6(b): If $S \neq \phi$ and Let r be the number of pairs in set S.

If $Z_i > Z_k$ *and* $T_i > T_k \quad (Z_i, T_i) \in S \ \forall i = 1, 2, ..., r$

Then (Z_i, T_i) will be replaced by (Z_k, T_k) and go to step 7.

Step6(c): If $Z_i > Z_k$ *and* $T_k > T_i$ *or* $Z_i < Z_k$ *and* $T_k < T_i$

Then consider both set (Z_i, T_i) and (Z_k, T_k) and go to step 7.

Step6(d): If $Z_i < Z_k$ *and* $T_i < T_k$

Then (Z_k, T_k) will not consider in S and go to step 7.

Step7: To check the total cost corresponds to basis is optimal or not.

Step7(a): Calculate $(c_{ij} - u_i - v_j)^k$ for all $i, j \notin B$ where u_i, v_j are the dual variables for i=1,2,. . . ,m,$m+1$; j=1,2,. . .,n, n+1.

Step7(b): Find $A_{ij}^{\ k} = (\mathrm{c_{ij}} - \mathrm{u_i} - \mathrm{v_j})^{\mathrm{k}} \times (\theta_{ij})^k$ for all $i, j \notin B$

Where $A_{ij}^{\ k}$ is the change in cost which occurs for introducing a non-basic cell (i, j) with value $(\theta_{ij})^k$ into the basis by making reallocation.

Step7(c): Calculate the fixed cost corresponding to non- basic cell.

$$F^k(\mathit{difference}) = F^k(NB) - F^k(\mathit{current})$$

Where $F_{ij}^{\ k}$(NB) is the total fixed cost involved for introducing the variable x_{ij} with values $(\theta_{ij})_k$ for all $i, j \notin B$ into the current basis to form a new basis.

Step7(e): $\Delta_{ij}^{\ k} = A_{ij}^{\ k} + F^k(\mathit{difference}) \ \forall (i, j) \notin B \ \mathit{and}$

$$T^k = \max\{t_{ij} \mid x_{ij} > 0\} \forall (i, j) \notin B.$$

Repeat the Step 6 only.

Step8(i): If $(\Delta_{ij})^k \leq 0$

Then most negative variable enter into basis and go to step3.

Step8(ii): Otherwise $(\Delta_{ij})^k > 0$ and Calculate $T = \max\{\ t_{ij} \mid x_{ij} > 0\ \} \quad \forall (i, j) \in B$

Then Block cells which time is 'T^k' or more than 'T^k'

Step8(ii)(a): If solution feasible then go to step2.

Otherwise solution infeasible then stops.

NUMARICAL EXAMPLE

Table 1 gives the values of variable cost C_{ij} (i=1, 2, 3; j=1, 2, 3) and Table 2 gives the values of time t_{ij} (i=1, 2, 3; j=1, 2, 3). The fixed costs are:

F_{11}=100, F_{12}=50, F_{13}=50

F_{21}=150, F_{22}=50, F_{23}=50

F_{31}=200, F_{32}=100, F_{33}=50

Table 1

Destination j → Origin i ↓	1	2	3	a_i
1	5	9	9	19
2	4	6	2	10
3	2	1	1	11
b_j	5	8	15	

Table 2

Destination j → Origin i ↓	1	2	3	a_i
1	15	8	2	19
2	10	13	11	10
3	6	9	17	11
b_j	5	8	15	

The total cost which is to be minimized is given by

$$\sum_{i=1}^{3}\sum_{j=1}^{3} C_{ij}x_{ij} + \sum_{i=1}^{3} F_i$$

where $$F_i = \sum_{l=1}^{3} \delta_{il} F_{il}\ , \quad i = 1,2,3$$

$$\delta_{i1} = 1, if \ \sum_{j=1}^{3} x_{ij} \ \geq 0, \ i = 1,2,3,$$

$$= 0, otherwise;$$

$$\delta_{i2} = 1, if \ \sum_{j=1}^{3} x_{ij} \geq 7, \ i = 1,2,3,$$

$$= 0, otherwise.$$

$$\delta_{i3} = \ 1, \ if \ \sum_{j=1}^{3} x_{ij} \ \geq 10, \ \ i = 1, 2,3,$$

$$= 0, \ otherwise.$$

Here F_i(i =1,2, 3) has three steps. Introducing a dummy destination $j=4$ and dummy source i=4 with zero cost means restricted flow in Table 1 and Table 2, we get Table 3 and Table 4 respectively.

Table 3

Destination j → Origin i ↓	1	2	3	4	a_i
1	5	9	9	0	19
2	4	6	2	0	10
3	2	1	1	0	11
4	0	0	0	M	3
b_j	5	8	15	12	

Table 4

Destination j → Origin i ↓	1	2	3	4	a_i
1	15	8	2	0	19
2	10	13	11	0	10
3	6	9	14	0	11
4	0	0	0	M	3
b_j	5	8	15	12	

A basic feasible solution of problem (P_1) is given in Table 5.

The right hand side value of Table 5 gives the total fixed cost of the current solution.

Table 5

Destination j → Origin i ↓	1	2	3	4	F^1(current)
1	5 (4)	9	9	0 (15)	100
2	4	6	2 (10)	0	200
3	2	1 (6)	1 (5)	0	350
4	0 (1)	0 (2)	0	M	

Total Cost = 701, Time =15

S = {(701, 15)}

Applying Step 7(a), we get $(C_{ij} - u_i - v_j)^1$ values, for all $i, j \notin B$ which are given in Table 6.

Table 6

(i, j)	(1,2)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(3,4)	(4,3)
$(C_{ij} - u_i - v_j)^1$	4	4	2	4	3	1	4	0

Applying Step 7(b), we get the values of A_{ij}^{1}, which are displayed in Table 7

Table 7

(i, j)	(1,2)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(3,4)	(4,3)
A_{ij}^{1}	8	8	2	24	3	1	4	0

Applying Step 7(c), we get the following results which are tabulated in Table 8.

Table 8

(i, j)	(1,2)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(3,4)	(4,3)
1	100	100	100	100	100	100	100	100
2	200	200	200	200	200	200	200	200
3	350	350	350	350	350	350	300	350
$F_{ij}(NB)$	650	650	650	650	650	650	600	650
$F_{ij}(Difference)$	0	0	0	0	0	0	-50	0

Applying Step 7(d), we get the values of Δ_{ij}^{1}, which are displayed in Table 9.

Table 9

(i, j)	(1,2)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(3,4)	(4,3)
Δ_{ij}^{1}	8	8	2	24	3	1	-46	0
t_{ij}^{1}	15	15	15	15	15	15	15	15

In Table 9, we see that

$$\text{Min}\ \{ \Delta_{ij}^{1},\ \Delta_{ij}^{1} < 0,\ i, j \notin B \} = -46 \quad \text{at}\ (3,4)\ \text{cell .applying step 8(i).}$$

Therefore, the variable to enter the basis is x_{34} and the new solution is given in Table 10.

Table 10

Destination j → Origin i ↓	1	2	3	4	F^1(current)
1	5 (5)	9	9	0 (14)	100
2	4	6	2 (10)	0	200
3	2	1 (5)	1 (5)	0 (1)	300
4	0	0 (3)	0	M	

Total Cost = 655, Time = 15

Applying Step 7(a), we get the following results as shown in Table 11.

Table 11

(i ,j)	(1,2)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(4,1)	(4,3)
$(C_{ij}- u_i- v_j)^2$	8	8	-2	4	-1	-3	-4	0

Applying Step 7(b), we get the values of A_{ij}^2, which are given in Table 12.

Table 12

(i ,j)	(1,2)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(4,1)	(4,3)
A_{ij}^2	40	40	-2	20	-1	-3	-4	0

Applying Step 7(c), the following results are obtained which are tabulated in Table 13.

Table 13

(i ,j)	(1,2)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(4,1)	(4,3)
1	150	150	100	100	100	100	100	100
2	200	200	200	200	200	200	200	200
3	200	200	350	300	350	350	350	300
$F_{ij}(NB)$	550	550	650	600	650	650	650	600
$F_{ij}(Difference)$	-50	-50	50	0	50	50	50	0

Applying Step 7(d), we get the values of Δ_{ij}^{2} which will be displayed in Table 14.

Table 14

(i ,j)	(1,2)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(4,1)	(4,3)
Δ_{ij}^{2}	-10	-10	48	20	49	47	46	0
t_{ij}^{2}	15	15	15	15	15	15	15	15

Here, Δ_{ij}^{2} is negative at (1, 2) cell. Therefore, entering x_{12} into the basis, we get the following solution in Table 15.

S = {(645, 15)}

Table15

Destination *j* → Origin *i*↓	1	2	3	4	F^{1}(current)
1	5 (5)	9 (5)	9	0 (4)	150
2	4	6	2 (10)	0	200
3	2	1	1 (5)	0 (6)	200
4	0	0 (3)	0	M	

Total Cost = 645, Time = 15

Applying Step 7(a), we get the following results in Table 16.

Table 16

(*i*, *j*)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(3,2)	(4,1)	(4,2)
$(C_{ij}- u_i- v_j)^3$	8	-2	-4	-1	-3	-8	4	8

Applying Step 7(b), we get the values of A_{ij}^{3} which are tabulated in Table 17.

Table 17

(i, j)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(3,2)	(4,1)	(4,2)
A_{ij}^3	40	-10	-20	-6	-15	-40	12	24

Applying Step 7(c), we get the following results which are displayed in Table 18.

Table 18

(i, j)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(3,2)	(4,1)	(4,2)
1	200	100	100	100	100	100	150	200
2	200	200	200	200	200	200	200	200
3	0	300	300	350	300	300	200	200
$F_{ij}(NB)$	400	600	600	600	600	600	550	600
$F_{ij}(Difference)$	-150	50	50	50	50	50	0	50

Applying Step 7(d), we get the following values of Δ_{ij}^3 which are tabulated in Table 19.

Table 19

(i, j)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(3,2)	(4,1)	(4,2)
Δ_{ij}^3	-110	40	30	44	35	-40	12	74
t_{ij}^3	15	14	15	15	14	15	15	15

$$\text{Min } \{ \Delta_{ij}^3,\ \Delta_{ij}^3 < 0,\ i, j \notin B \} = -110 \quad \text{at } (1, 3) \text{ cell}.$$

Therefore, the variable to enter the basis is x_{13} and the new solution is given in Table 20

S = {(535, 15), (680,14)}

Table 20

Destination j → Origin i ↓	1	2	3	4	F^1(current)
1	5 (5)	9 (5)	9 (5)	0 (4)	200
2	4	6	2 (10)	0	200
3	2	1	1	0 (11)	0
4	0	0 (3)	0	M	

Total Cost = 535, Time = 15

Applying Step 7(a), we get $(C_{ij} - u_i - v_j)^4$ values, for all $i, j \notin B$ which are given in Table 21.

Table 21

(i, j)	(2,1)	(2,2)	(2,4)	(3,1)	(3,2)	(3,3)	(4,1)	(4,3)
$(C_{ij} - u_i - v_j)^4$	6	4	7	-3	-8	-8	4	0

Applying Step 7(b), we get the values of A_i^4, which are displayed in Table 22.

Table 22

(i, j)	(2,1)	(2,2)	(2,4)	(3,1)	(3,2)	(3,3)	(4,1)	(4,3)
A_{ij}^4	30	20	28	-15	-40	-40	12	0

Applying Step 7(c), we get the following results which are tabulated in Table 23.

Table 23

(i, j)	(2,1)	(2,2)	(2,4)	(3,1)	(3,2)	(3,3)	(4,1)	(4,3)
1	200	200	200	150	150	150	200	200
2	200	200	150	200	200	200	200	200
3	0	0	0	200	200	200	0	0
$F_{ij}(NB)$	400	400	350	550	550	550	400	400
$F_{ij}(Difference)$	0	0	-50	150	150	150	0	0

Applying Step 7(d), we get the values of Δ_{ij}^4, which are displayed in Table 24.

Table 24

(i, j)	(2,1)	(2,2)	(2,4)	(3,1)	(3,2)	(3,3)	(4,1)	(4,3)
Δ_{ij}^4	30	20	-22	135	110	110	12	0
t_{ij}^4	11	15	15	11	15	15	15	15

we see that

$$\text{Min}(\Delta_{ij}^4,\ \Delta_{ij}^4 < 0,\ i, j \notin B\} = -22 \quad \text{at } (2,4) \text{ cell}.$$

Therefore, the variable to enter the basis is x_{24} and the new solution is given in Table 25.

$$S = \{(513, 15), (565, 11)\}$$

Table 25

Destination j → Origin i ↓	1	2	3	4	F^1(current)
1	5 (5)	9 (5)	9 (9)	0	200
2	4	6	2 (6)	0(4)	150
3	2	1	1	0 (11)	0
4	0	0(3)	0	M	

Total cost = 513,   Time = 15

Applying Step 7(a), we get the following results in Table 26.

Table 26

(i, j)	(1,4)	(2,1)	(2,2)	(3,1)	(3,2)	(3,3)	(4,1)	(4,3)
$(C_{ij}- u_i- v_j)^5$	-7	6	4	4	-1	-1	4	0

Applying Step 7(b), we get the values of A_{ij}^5, which are given in Table 27.

Table 27

(i, j)	(1,4)	(2,1)	(2,2)	(3,1)	(3,2)	(3,3)	(4,1)	(4,3)
A_{ij}^5	-28	30	20	20	-5	-6	12	0

Applying Step 7(c), the following results are obtained which are tabulated in Table 28.

Table 28

(i, j)	(1,4)	(2,1)	(2,2)	(3,1)	(3,2)	(3,3)	(4,1)	(4,3)
1	200	200	200	200	200	200	200	200
2	200	150	150	150	150	0	150	150
3	0	0	0	200	200	200	0	0
$F_{ij}(NB)$	400	350	350	550	550	400	350	350
$F_{ij}(Difference)$	50	0	0	200	200	50	0	0

Applying Step 7(d), we get the values of Δ_{ij}^5 which are displayed in Table 29.

Table 29

(i, j)	(1,4)	(2,1)	(2,2)	(3,1)	(3,2)	(3,3)	(4,1)	(4,3)
Δ_{ij}^5	23	30	20	220	195	44	12	0
t_{ij}^5	15	15	15	11	15	15	15	15

All the values of $\Delta_{ij}^5 > 0$, So 513 is optimal cost.

S = {(513, 15), (543,11)}

Table 31

Destination j → Origin i ↓	1	2	3	4	F^1(current)
1	M	9	9 (4)	0 (15)	100
2	4	6	2 (10)	0	200
3	2 (2)	1 (8)	1 (1)	0	300
4	0 (3)	0	0	M	

Total cost = 719, Time = 14

Applying Step 7(a), we get the following results in Table 32.

Table 32

(i, j)	(1,2)	(2,1)	(2,2)	(2,4)	(3,4)	(4,2)	(4,3)
$(C_{ij} - u_i - v_j)^6$	0	1	4	7	8	1	1

Applying Step 7(b), we get the following results in Table 33.

Table 33

(i, j)	(1,2)	(2,1)	(2,2)	(2,4)	(3,4)	(4,2)	(4,3)
A_{ij}^6	0	2	32	70	8	3	1

Applying Step 7(c), we get the following results which are displayed in Table 34.

Table 34

(i, j)	(1,2)	(2,1)	(2,2)	(2,4)	(3,4)	(4,2)	(4,3)
1	100	100	100	200	100	100	100
2	200	250	200	0	200	200	200
3	350	300	350	350	300	350	350
$F_{ij}(NB)$	650	650	650	550	600	650	650
$F_{ij}(Difference)$	0	0	0	-100	-50	0	0

Applying Step 7(d), we get the following values of Δ_{ij}^{6} which are tabulated in Table 35.

Table 35

(i, j)	(1,2)	(2,1)	(2,2)	(2,4)	(3,4)	(4,2)	(4,3)
Δ_{ij}^{6}	0	2	32	-30	-42	3	1
t_{ij}^{6}	14	14	14	14	11	14	11

$$\text{Min}(\Delta_{ij}^{6},\ \Delta_{ij}^{6} < 0,\ i, j \notin B\} = -42 \quad \text{at } (3,4) \text{ cell .}$$

Therefore, the variable to enter the basis is x_{34} and the new solution is given in Table 36.

S = {(513, 15), (543,11)}

Table 36

Destination j → Origin i ↓	1	2	3	4	F^{1}(current)
1	M	9	9 (5)	0 (14)	100
2	4	6	2 (10)	0	200
3	2 (2)	1 (8)	1	0 (1)	300
4	0 (3)	0	0	M	

Total cost = 677, Time = 11

Applying Step 7(a), we get the following results in Table 37.

Table 37

(i, j)	(1,2)	(2,1)	(2,2)	(2,4)	(3,3)	(4,2)	(4,3)
$(C_{ij}-u_i-v_j)^7$	8	9	12	7	-8	-1	7

Applying Step 7(b), we get the following results in Table 38.

Table 38

(i, j)	(1,2)	(2,1)	(2,2)	(2,4)	(3,4)	(4,2)	(4,3)
A_{ij}^7	64	18	96	70	-8	-3	7

Applying Step 7(c), we get the following results in Table 39.

Table 39

(i, j)	(1,2)	(2,1)	(2,2)	(2,4)	(3,3)	(4,2)	(4,3)
1	200	100	200	200	100	100	100
2	200	200	200	0	200	200	200
3	200	300	200	300	350	300	350
$F_{ij}(NB)$	600	600	600	500	650	600	650
$F_{ij}(Difference)$	0	0	0	-100	50	0	50

Applying Step 7(d), we get the following values of Δ_{ij}^7 which are tabulated in Table 40.

Table 40

(i, j)	(1,2)	(2,1)	(2,2)	(2,4)	(3,3)	(4,2)	(4,3)
Δ_{ij}^7	64	18	96	-30	42	-3	57
t_{ij}^7	11	11	11	9	14	11	11

$$\text{Min}(\Delta_{ij}^7,\ \Delta_{ij}^7 < 0,\ i, j \notin B\} = -30 \quad \text{at } (2,4) \text{ cell .}$$

Therefore, the variable to enter the basis is x_{24} and the new solution is given in Table 41.

S = {(513, 15), (543,11), (647, 9)}

Table 41

Destination j → Origin i ↓	1	2	3	4	F^1(current)
1	M	9	9 (15)	0 (4)	200
2	4	6	2 (10)	0	0
3	2 (2)	1 (8)	1	0 (1)	300
4	0 (3)	0	0	M	

Total cost = 647 , Time = 11

Applying Step 7(a), we get the following results in Table 42.

Table 42

(i, j)	(1,2)	(2,1)	(2,2)	(2,3)	(3,3)	(4,2)	(4,3)
$(C_{ij} - u_i - v_j)^8$	8	2	5	-7	-8	1	-7

Applying Step 7(b), we get the following results in Table 43.

Table 43

(i, j)	(1,2)	(2,1)	(2,2)	(2,4)	(3,3)	(4,2)	(4,3)
A_{ij}^8	32	4	40	-70	-8	3	-7

Applying Step 7(c), we get the following results which are displayed in Table 44.

Table 44

(i, j)	(1,2)	(2,1)	(2,2)	(2,3)	(3,3)	(4,2)	(4,3)
1	200	200	200	100	200	200	200
2	0	150	200	200	0	0	0
3	200	200	200	300	350	300	350
$F_{ij}(NB)$	400	550	600	600	550	500	550
$F_{ij}(Difference)$	-100	50	100	100	50	0	50

Applying Step 7(d), we get the following values of Δ_{ij}^{8} which are tabulated in Table 45.

Table 46

(i, j)	(1,2)	(2,1)	(2,2)	(2,3)	(3,3)	(4,2)	(4,3)
Δ_{ij}^{8}	-68	54	140	30	42	3	43
t_{ij}^{8}	9	10	13	11	14	9	9

$$\text{Min}(\Delta_{ij}^{8},\ \Delta_{ij}^{8} < 0,\ i, j \notin B\} = -68 \quad \text{at } (1, 2) \text{ cell .}$$

Therefore, the variable to enter the basis is x_{12} and the new solution is given in Table 47.

$$S = \{(513, 15), (543,11), (579, 9)\}$$

Table 47

Destination j → Origin i ↓	1	2	3	4	F^1(current)
1	M	9 (4)	9 (15)	0	200
2	4	6	2	0 (10)	0
3	2 (2)	1 (4)	1	0 (5)	200
4	0 (3)	0	0	M	

Total cost = 579, Time = 9

Applying Step 7(a), we get the following results which are displayed in Table 48.

Table 48

(i, j)	(1,4)	(2,1)	(2,2)	(2,3)	(3,3)	(4,2)	(4,3)
$(C_{ij} - u_i - v_j)^9$	-8	2	5	1	0	1	1

Applying Step 7(b), we get the following results which are displayed in Table 49.

Table 49

(i, j)	(1,4)	(2,1)	(2,2)	(2,4)	(3,3)	(4,2)	(4,3)
A_{ij}^9	-32	4	20	4	0	3	3

Applying Step 7(c), we get the following results which are displayed in Table 50.

Table 50

(i, j)	(1,4)	(2,1)	(2,2)	(2,3)	(3,3)	(4,2)	(4,3)
1	200	200	200	200	200	200	200
2	0	150	150	150	0	0	0
3	300	200	200	200	200	200	200
$F_{ij}(NB)$	500	550	550	550	400	400	400
$F_{ij}(Difference)$	100	150	150	150	0	0	0

Applying Step 7(d), we get the following values of Δ_{ij}^9 which are tabulated in Table 51.

Table 51

(i, j)	(1,4)	(2,1)	(2,2)	(2,3)	(3,3)	(4,2)	(4,3)
Δ_{ij}^9	68	154	170	154	0	3	3
t_{ij}^9	9	10	13	11	14	9	9

All the $\Delta_{ij}^{9} \geq 0$, So (579, 9) is the second optimal solution.

S = { (513, 15), (543, 11), (579,9)}

After this iteration, we see that the solution is infeasible. Then we get three cost-time trade-off pairs as

{(513, 15), (543, 11), (579,9)}

CONCLUSION

Algorithm developed by Thirwani *et. al* (1997) for a fixed charge bi-criterion transportation problem with restricted flow. Find two pairs in cost time trade of {(513, 15), (579, 9)}while the modified algorithm is applied to same problem which produce three cost-time trade-off pairs {(513, 15), (543, 11), (579, 9)} and two of them is similar.

CHAPTER - 3

"MODIFIED FIXED CHARGE BI-CRITERION TRANSPORTATION PROBLEM WITH RESTRICTED FLOW"

MODIFIED FIXED CHARGE BI-CRITERION TRANSPORTATION PROBLEM WITH RESTRICTED FLOW

In chapter 2 (FCTP with restricted flow) in which a fixed cost called set up cost is incurred for every origin is considered. In this chapter it is considered that the set up cost is incurred for every cell and solution procedure is modified. The algorithm used in chapter 2 is applied to the same example to find the cost- time trade off pairs for FCTP with restricted flow.

3.1 SOLUTION PROCEDURE:

For the solution of (FCBTP), the algorithm given by Thirwani *et. al.* (1997) has been improved by using reoptimizing procedure and theoretical development given by Thiwani *et. al*. (1997) which is reported ready reference.

(FCBTP) is converted into two problems (P1) and (P2) where

(P1): Minimize$\left[\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij} + \sum_{i\in I} F_i\right]$

Subject to (2.1)

and

(P2): Minimize $[\max_{\substack{i\in I\\ j\in J}} [t_{ij} \,|x_{ij} > 0]]$

Subject to (2.1)

For formulation $F_i (i = 1, 2, \ldots, m)$ we assume that $F_i (i = 1, 2, \ldots, m)$ has m number of steps so that

$$F_i = \sum_{l=1}^{n} \delta_{il}\, F_{il}\,, \quad i = 1, 2, \ldots, m$$

Where $\delta_{il} = 1$ if $x_{ij} > 0, \quad i = 1, 2, \ldots, m;\ l = 1, 2, \ldots\ldots, n;$

= 0 otherwise

F_{il} ($i = 1, 2, \ldots, m;\ l = 1, 2, \ldots, n)$ is fixed cost to origin i to destination j .

The flow constraint in the problem (P1) implies that a total $\left(\sum_{i\in I} a_i - P\right)$ of source reserves has to be kept at the various sources and a total $\left(\sum_{j\in J} b_j - P\right)$ of destination slacks to be retained at the various destinations to receive the source reserves and an extra source to fill up the destination slacks are introduced. Hence, the related fixed charge bi-criterion transportation problem (RFCBTP) associated with fixed charge bi-criterion transportation problem (FCBTP) is

$$\text{Minimize}\left\{\sum_{i\in I'}\sum_{j\in J'} c'_{ij} y_{ij} + \sum_{i\in I} F_i\,,\ \text{Max}[t'_{ij}\,|y_{ij} > 0]\right\}$$

Subject to

$$\left.\begin{aligned}
&\sum_{j\in J'} y_{ij} = a'_i, && i\in I' \\
&\sum_{i\in I'} y_{ij} = b'_j, && j\in J' \\
&\sum_{i\in I'}\sum_{j\in J'} y_{ij} = P && \\
&\quad y_{ij}\geq 0,\ i\in I',\ j\in J' &&
\end{aligned}\right\} \qquad (3.2)$$

$$where\ \ I' = \{1, 2, \ldots, m+1\} = I \cup \{m+1\}$$
$$J' = \{1, 2, \ldots, n+1\} = J \cup \{n+1\}$$

$$a'_i = a_i,\ \ i \in I,\ a'_{m+1} = \left(\sum_{j \in J} b_j - P \right)$$
$$b'_j = b_j,\ \ \ j \in J,\ b'_{n+1} = \left(\sum_{i \in I} a_i - P \right)$$
$$c'_{ij} = c_{ij}(i, j) \in I \times J$$
$$t'_{ij} = t_{ij}(i, j) \in I \times J$$
$$c'_{i,n+1} = c'_{m+1,j} = 0,\ \ i \in I,\ \ j \in J$$
$$t'_{i,n+1} = t'_{m+1,j} = 0,\ \ i \in I,\ \ j \in J$$
$$F_{m+1} = 0,$$
$$c'_{m+1,n+1} = M, t'_{m+1,n+1} > \underset{\substack{i \in I' \\ j \in J'}}{Max}[t_{ij} \mid y_{ij} > 0]$$

Where M is a large positive number.

(RFCBTP) is separated into two problems (RP1) and (RP2).

(RP1): Minimize $\left\{\sum_{i \in I'} \sum_{j \in J'} c'_{ij} y_{ij} + \sum_{i \in I} F_i\right\}$

Subject to (3.2)

(RP2): Minimize $T = \underset{\substack{i \in I' \\ j \in J'}}{Max} [t_{ij} | y_{ij} > 0]$

Subject to (3.2)

Thirwani *et. al*. (1997) developed some theory related to this problem and is given below.

NUMARICAL EXAMPLE

Table 1 gives the values of variable cost C_{ij} (i=1, 2, 3; j=1, 2, 3) and Table 2 gives the values of time t_{ij} (i=1, 2, 3; j=1, 2, 3). The fixed costs for every cell are:

F_{11}=100, F_{12}=50, F_{13}=50

F_{21}=150, F_{22}=50, F_{23}=50

F_{31}=200, F_{32}=100, F_{33}=50

Table 1

Destination j → Origin i ↓	1	2	3	a_i
1	5	9	9	19
2	4	6	2	10
3	2	1	1	11
b_j	5	8	15	

Table 2

Destination j → Origin i ↓	1	2	3	a_i
1	15	8	2	19
2	10	13	11	10
3	6	9	17	11
b_j	5	8	15	

The total cost which is to be minimized is given by

$$\sum_{i=1}^{3}\sum_{j=1}^{3} C_{ij} x_{ij} + \sum_{i=1}^{3} F_i$$

$$F_i = \sum_{l=1}^{3} \delta_{il}\, F_{il}\,, \quad i = 1, 2, 3.$$

Where $\delta_{il} = 1$ if $x_{ij} > 0, \quad i = 1,2,3;\ l = 1,2,3;$

= 0 otherwise

Here F_i(i=1,2, 3) has three steps. Introducing a dummy destination $j=4$ and dummy source i=4 with zero cost means restricted flow in Table 1 and Table 2, we get Table 3 and Table 4 respectively.

Table 3

Destination $j\rightarrow$ Origin $i\downarrow$	1	2	3	4	a_i
1	5	9	9	0	19
2	4	6	2	0	10
3	2	1	1	0	11
4	0	0	0	M	3
b_j	5	8	15	12	

Table 4

Destination $j\rightarrow$ Origin $i\downarrow$	1	2	3	4	a_i
1	15	8	2	0	19
2	10	13	11	0	10
3	6	9	14	0	11
4	0	0	0	M	3
b_j	5	8	15	12	

A basic feasible solution of problem (P_1) is given in Table 5.

The right hand side value of Table 5 gives the total fixed cost of the current solution.

Table 5

Destination $j\rightarrow$ Origin $i\downarrow$	1	2	3	4	F^1(current)
1	5 (4)	9	9	0 (15)	100
2	4	6	2 (10)	0	50
3	2	1 (6)	1 (5)	0	150
4	0 (1)	0 (2)	0	M	

Total Cost = 351, Time =15

S = {(351, 15)}

Applying Step 7(a), we get $(C_{ij}– u_i - v_j)^1$ values, for all $i, j \notin B$ which are given in Table 6.

Table 6

(i, j)	(1,2)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(3,4)	(4,3)
$(C_{ij}– u_i- v_j)^1$	4	4	2	4	3	1	4	0

Applying Step 7(b), we get the values of A_{ij}^{1}, which are displayed in Table 7

Table 7

(i, j)	(1,2)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(3,4)	(4,3)
A_{ij}^{1}	8	8	2	24	3	1	4	0

Applying Step 7(c), we get the following results which are tabulated in Table 8.

Table 8

(i, j)	(1,2)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(3,4)	(4,3)
1	150	150	100	100	100	100	100	100
2	50	50	200	100	50	50	50	50
3	150	150	150	50	150	350	150	150
$F_{ij}(NB)$	350	350	450	250	300	500	300	300
$F_{ij}(Difference)$	50	50	150	-50	0	200	0	0

Applying Step 7(d), we get the values of Δ_{ij}^{1}, which are displayed in Table 9.

Table 9

(i, j)	(1,2)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(3,4)	(4,3)
Δ_{ij}^{1}	58	58	152	-26	3	201	4	0
t_{ij}^{1}	15	15	15	15	15	15	15	15

In Table 9, we see that

$$\text{Min}\ \{\ \Delta_{ij}^{1},\ \Delta_{ij}^{1} < 0,\ i, j \notin B\ \} = -26 \quad \text{at (2,2) cell .}$$

By applying step 8(i).Therefore, the variable to enter the basis is x_{22} and the new solution is given in Table 10.

S = {(325, 15)}

Table 10

Destination j→ Origin i↓	1	2	3	4	F^1(current)
1	5 (4)	9	9	0 (15)	100
2	4	6 (6)	2 (4)	0	100
3	2	1	1 (11)	0	50
4	0 (1)	0 (2)	0	M	

Total Cost = 325, Time = 15

Applying Step 7(a), we get the following results as shown in Table 11.

Table 11

(i, j)	(1,2)	(1,3)	(2,1)	(2,4)	(3,1)	(3,2)	(3,4)	(4,3)
$(C_{ij} - u_i - v_j)^2$	4	8	-2	-1	-3	-4	0	4

Applying Step 7(b), we get the values of A_{ij}^2, which are given in Table 12.

Table 12

(i, j)	(1,2)	(1,3)	(2,1)	(2,4)	(3,1)	(3,2)	(3,4)	(4,3)
A_{ij}^2	8	16	-2	-1	-3	-24	0	8

Applying Step 7(c), the following results are obtained which are tabulated in Table 13.

Table 13

(i, j)	(1,2)	(1,3)	(2,1)	(2,4)	(3,1)	(3,2)	(3,4)	(4,3)
1	150	150	100	100	100	100	100	100
2	100	100	250	100	100	50	100	100
3	50	50	50	50	250	150	50	50
$F_{ij}(NB)$	300	300	400	250	450	300	250	250
$F_{ij}(Difference)$	50	50	100	0	150	50	0	0

Applying Step 7(d), we get the values of Δ_{ij}^2 which are displayed in Table 14.

Table 14

(i, j)	(1,2)	(1,3)	(2,1)	(2,2)	(2,4)	(3,1)	(3,4)	(4,3)
Δ_{ij}^2	58	66	98	-1	147	26	0	8
t_{ij}^2	15	15	15	15	15	15	15	15

In Table 14, we see that

$$\text{Min } \{\Delta_{ij}^2, \Delta_{ij}^2 < 0,\ i, j \notin B\} = -1 \quad \text{at } (2,4) \text{ cell} .$$

Here Δ_{ij}^2 is negative at (2, 4) cell. By applying step 8(i).Therefore, entering x_{24} into the basis, we get the following solution in Table 15.

Table 15

Destination $j\rightarrow$ Origin $i\downarrow$	1	2	3	4	F^1(current)
1	5 (5)	9	9	0 (14)	100
2	4	6 (5)	2 (4)	0 (1)	100
3	2	1	1 (11)	0	50
4	0	0 (3)	0	M	

Total Cost = 324, Time = 15

S = {(324, 15)}

Applying Step 7(a), we get the following results in Table 16.

Table 16

(i, j)	(1,2)	(1,3)	(2,1)	(3,1)	(3,2)	(3,4)	(4,1)	(4,3)
$(C_{ij}- u_i- v_j)^3$	3	7	-1	-2	-4	1	-5	-2

Applying Step 7(b), we get the values of A_{ij}^3 which are tabulated in Table 17.

Table 17

(i, j)	(1,2)	(1,3)	(2,1)	(3,1)	(3,2)	(3,4)	(4,1)	(4,3)
A_{ij}^3	15	28	-1	-2	-20	1	-5	-6

Applying Step 7(c), we get the following results which are displayed in Table 18.

Table 18

(i, j)	(1,2)	(1,3)	(2,1)	(3,1)	(3,2)	(3,4)	(4,1)	(4,3)
1	150	150	100	100	100	100	100	100
2	50	50	250	100	50	100	100	100
3	50	50	50	250	150	50	50	50
$F_{ij}(NB)$	250	250	400	450	300	250	250	250
$F_{ij}(Difference)$	0	0	150	150	50	0	0	0

Applying Step 7(d), we get the following values of Δ_{ij}^3 which are tabulated in Table 19.

Table 19

(i, j)	(1,2)	(1,3)	(2,1)	(3,1)	(3,2)	(3,4)	(4,1)	(4,3)
Δ_{ij}^{3}	15	28	149	148	30	1	-5	-6
t_{ij}^{3}	15	15	15	15	15	15	15	15

In Table 9, we see that

$$\text{Min } \{\Delta_{ij}^{3}, \Delta_{ij}^{3} < 0,\ i, j \notin B\} = -6 \quad \text{at } (4,3) \text{ cell .}$$

Therefore, the variable to enter the basis is x_{13} and the new solution is given in Table 20

S = {(318, 15)}

Table 20

Destination $j \rightarrow$ Origin $i \downarrow$	1	2	3	4	F^{1}(current)
1	5 (5)	9	9	0 (14)	100
2	4	6 (8)	2 (1)	0 (1)	100
3	2	1	1 (11)	0	50
4	0	0	0 (3)	M	

Total Cost = 318, Time = 15

Applying Step 7(a), we get $(C_{ij} - u_i - v_j)^4$ values, for all $i, j \notin B$ which are given in Table 21.

Table 21

(i, j)	(1,2)	(1,3)	(2,1)	(3,1)	(3,2)	(3,4)	(4,1)	(4,2)
$(C_{ij} - u_i - v_j)^4$	3	8	-1	-3	-5	0	-4	-5

Applying Step 7(b), we get the values of A_i^4, which are displayed in Table 22

Table 22

(i, j)	(1,2)	(1,3)	(2,1)	(3,1)	(3,2)	(3,4)	(4,1)	(4,2)
A_{ij}^4	24	8	-1	-3	-40	0	-4	-15

Applying Step 7(c), we get the following results which are tabulated in Table 23.

Table 23

(i, j)	(1,2)	(1,3)	(2,1)	(3,1)	(3,2)	(3,4)	(4,1)	(4,2)
1	150	150	100	100	100	100	100	100
2	50	50	250	100	50	100	100	100
3	50	50	50	250	150	50	50	50
$F_{ij}(NB)$	250	250	400	450	300	250	250	250
$F_{ij}(Difference)$	0	0	150	200	50	0	0	0

Applying Step 7(d), we get the values of Δ_{ij}^4, which are displayed in Table 24.

Table 24

(i, j)	(1,2)	(1,3)	(2,1)	(3,1)	(3,2)	(3,4)	(4,1)	(4,2)
Δ_{ij}^4	24	8	149	197	10	0	-4	-15
t_{ij}^4	15	15	15	15	15	15	15	15

In Table 24, we see that

$$\text{Min } \{\Delta_{ij}^4, \Delta_{ij}^4 < 0,\ i, j \notin B\} = -15 \text{ at } (4, 2) \text{ cell .}$$

Table 25

Destination $j\rightarrow$ Origin $i\downarrow$	1	2	3	4	F^1(current)
1	5 (5)	9	9	0 (14)	100
2	4	6 (5)	2 (4)	0 (1)	100
3	2	1	1 (11)	0	50
4	0	0 (3)	0	M	

Total Cost = 303, Time = 15

Applying Step 7(a), we get the following results as shown in Table 26.

Table 26

(i, j)	(1,2)	(1,3)	(2,1)	(3,1)	(3,2)	(3,4)	(4,1)	(4,3)
$(C_{ij} - u_i - v_j)^5$	3	7	-1	-2	-4	1	1	4

Applying Step 7(b), we get the values of A_{ij}^1, which are displayed in Table 27.

Table 27

(i, j)	(1,2)	(1,3)	(2,1)	(3,1)	(3,2)	(3,4)	(4,1)	(4,3)
A_{ij}^5	15	28	-1	-2	-20	1	1	12

Applying Step 7(c), we get the following results which are tabulated in Table 28.

Table 28

(i, j)	(1,2)	(1,3)	(2,1)	(3,1)	(3,2)	(3,4)	(4,1)	(4,3)
1	150	150	100	100	100	100	100	100
2	50	50	100	50	150	100	100	100
3	50	50	250	150	150	50	50	50
$F_{ij}(NB)$	250	250	450	300	300	250	250	250
$F_{ij}(Difference)$	0	0	200	50	50	0	0	0

Applying Step 7(d), we get the values of Δ_{ij}^5, which are displayed in Table 29.

Table 29

(i, j)	(1,2)	(1,3)	(2,1)	(3,1)	(3,2)	(3,4)	(4,1)	(4,3)
Δ_{ij}^5	15	28	199	48	30	1	1	12
t_{ij}^5	15	15	15	15	15	15	15	15

We see $\Delta_{ij}^5 > 0$. Applying step 8(ii).No non-basic cell enter into the basis. The optimal cost is 303 and time 15. Now block the cells where time is 15 or more than 15

Table 30

Destination j→ Origin i↓	1	2	3	4	F^1(current)
1	M	9	9 (4)	0 (15)	50
2	4	6	2 (10)	0	50
3	2 (2)	1 (8)	1 (1)	0	350
4	0 (3)	0	0	M	

Total Cost = 519, Time = 14

S = {(303, 15), (519, 14)}

Applying Step 7(a), we get the following results as shown in Table 31.

Table 31

(i, j)	(1,2)	(2,1)	(2,2)	(2,4)	(3,4)	(4,2)	(4,3)
$(C_{ij} - u_i - v_j)^6$	0	1	4	7	8	1	1

Applying Step 7(b), we get the values of A_{ij}^6, which are displayed in Table 32.

Table 32

(i, j)	(1,2)	(2,1)	(2,2)	(2,4)	(3,4)	(4,2)	(4,3)
A_{ij}^6	0	2	32	70	8	3	1

Applying Step 7(c), we get the following results which are tabulated in Table 33.

Table 33

(i, j)	(1,2)	(2,1)	(2,2)	(2,4)	(3,4)	(4,2)	(4,3)
1	50	50	50	50	50	50	50
2	50	200	100	0	50	50	50
3	350	150	250	350	300	350	300
$F_{ij}(NB)$	450	400	400	400	400	450	400
$F_{ij}(Difference)$	0	-50	-50	-50	-50	0	-50

Applying Step 7(d), we get the values of Δ_{ij}^6, which are displayed in Table 34.

Table 34

(i, j)	(1,2)	(2,1)	(2,2)	(2,4)	(3,4)	(4,2)	(4,3)
Δ_{ij}^6	0	-48	-18	-40	-42	3	-49
t_{ij}^6	14	14	14	14	14	14	11

In Table 34, we see that

$$\text{Min}\ \{\Delta_{ij}^{6},\ \Delta_{ij}^{6} < 0,\ i, j \notin B\} = -49 \quad \text{at}\ \ (4, 3)\ \text{cell}.$$

$$S = \{(303, 15), (470, 11)\}$$

Table 35

Destination $j\rightarrow$ Origin $i\downarrow$	1	2	3	4	F^{1}(current)
1	M	9	9 (4)	0 (15)	50
2	4	6	2 (10)	0	50
3	2 (3)	1 (8)	1	0	300
4	0 (2)	0	0 (1)	M	

Total Cost = 470, Time = 11

Applying Step 7(a), we get the following results as shown in Table 36.

Table 36

(i, j)	(1,2)	(2,1)	(2,2)	(2,4)	(3,3)	(3,4)	(4,2)
$(C_{ij} - u_i - v_j)^7$	1	2	5	7	-1	7	1

Applying Step 7(b), we get the values of A_{ij}^{7}, which are displayed in Table 37.

Table 37

(i, j)	(1,2)	(2,1)	(2,2)	(2,4)	(3,3)	(3,4)	(4,2)
A_{ij}^{7}	2	4	10	70	-1	7	2

Applying Step 7(c), we get the following results which are tabulated in Table 38.

Table 38

(*i*, *j*)	(1,2)	(2,1)	(2,2)	(2,4)	(3,3)	(3,4)	(4,2)
1	100	50	50	50	50	50	50
2	50	200	100	0	50	50	50
3	350	300	300	300	350	300	300
$F_{ij}(NB)$	500	550	450	350	450	400	400
$F_{ij}(Difference)$	100	150	50	-50	50	0	0

Applying Step 7(d), we get the values of Δ_{ij}^1, which are displayed in Table 39.

Table 39

(*i*, *j*)	(1,2)	(2,1)	(2,2)	(2,4)	(3,3)	(3,4)	(4,2)
Δ_{ij}^7	102	154	60	20	49	7	2
t_{ij}^7	11	11	13	9	14	11	11

We see $\Delta_{ij}^7 > 0$. No non-basic cell enter into the basis. The second cost-time trade off pair is (470, 11) Now block the cells where time is 11 or more than 11.

S = {(303, 15), (470, 11), (490, 9)}

Table 40

Destination *j*→ Origin *i*↓	1	2	3	4	F^1(current)
1	M	9 (2)	9 (12)	0 (5)	100
2	4	M	M	0 (10)	0
3	2 (5)	1 (6)	M	0	300
4	0	0	0 (3)	M	

Total Cost = 542, Time = 9

Applying Step 7(a), we get the following results as shown in Table 41.

Table 41

(i, j)	(2,1)	(3,4)	(4,1)	(4,2)
$(C_{ij}- u_i- v_j)^8$	-6	8	-1	0

Applying Step 7(b), we get the values of A_{ij}^{8}, which are displayed in Table 42

Table 42

(i, j)	(2,1)	(3,4)	(4,1)	(4,2)
A_{ij}^{8}	-12	40	-2	0

Applying Step 7(c), we get the following results which are tabulated in Table 43.

Table 43

(i, j)	(2,1)	(3,4)	(4,1)	(4,2)
1	50	100	50	50
2	150	0	0	0
3	300	300	300	300
$F_{ij}(NB)$	500	400	350	350
$F_{ij}(Difference)$	100	0	-50	-50

Applying Step 7(d), we get the values of Δ_{ij}^{8}, which are displayed in Table 44.

Table 44

(i, j)	(2,1)	(3,4)	(4,1)	(4,2)
Δ_{ij}^{8}	88	40	-52	-50
t_{ij}^{8}	10	9	9	9

In Table 44, we see that

$$\text{Min } \{\Delta_{ij}^{8}, \Delta_{ij}^{8} < 0,\ i, j \notin B\} = -52 \quad \text{at } (4,1) \text{ cell .}$$

S = {(303, 15), (470, 11), (490, 9)}

Table 45

Destination $j\rightarrow$ Origin $i\downarrow$	1	2	3	4	F^1(current)
1	M	9	9 (14)	0 (5)	50
2	4	M	M	0 (10)	0
3	2 (3)	1 (8)	M	0	300
4	0 (2)	0	0 (1)	M	

Total Cost = 490, Time = 9

Applying Step 7(a), we get the following results as shown in Table 46.

Table 46

(i, j)	(1,2)	(2,1)	(3,4)	(4,2)
$(C_{ij}- u_i- v_j)^9$	1	-5	7	9

Applying Step 7(b), we get the values of A_{ij}^9, which are displayed in Table 47

Table 47

(i, j)	(1,2)	(2,1)	(3,4)	(4,2)
A_{ij}^9	2	-10	7	18

Applying Step 7(c), we get the following results which are tabulated in Table 48.

Table 48

(i, j)	(1,2)	(2,1)	(3,4)	(4,2)
1	100	50	50	50
2	0	150	0	0
3	300	300	300	300
$F_{ij}(NB)$	400	500	350	350
$F_{ij}(Difference)$	50	150	0	0

Applying Step 7(d), we get the values of Δ_{ij}^{9}, which are displayed in Table 49.

Table 49

(i, j)	(1,2)	(2,1)	(3,4)	(4,2)
Δ_{ij}^{9}	52	140	7	18
t_{ij}^{9}	9	10	11	9

We see $\Delta_{ij}^{9} > 0$. No non-basic cell enter into the basis. The third cost-time trade off pair is (490, 9) Now block the cells where time is 9 or more than 9.

S = {(303, 15), (470, 11), (490, 9)}

Table 50

Destination $j\rightarrow$ Origin $i\downarrow$	1	2	3	4
1	M	9	9	0
2	M	M	M	0
3	2	M	M	0
4	0	0	0	M

Now the allocation is not possible. Solution is infeasible.

Result is {(303, 15), (470, 11), (490, 9)}

CONCLUSION

The fixed charge bi-criterion transportation problem with restricted flow given by Thirwan*et. al.*(1997) is modified. In this problem it is considered that the fixed cost is incurred. Three time-cost trades off pairs are obtained for the example considered in chapter 2.

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