STOCHASTIC OPTIMAL POWER GENERATION SCHEDULING

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Dedicated

to

my Father

C E R T I F I C A T E

Certified that the thesis entitled, "Stochastic Optimal Power Generation Scheduling" which is being submitted by Shri Satish Chandra Parti, in fulfilment of the requirements for the award of the Degree of Doctor of Philosophy in Electrical Engineering, Thapar Institute of Engineering & Technology (Deemed University), Patiala is a record of candidate's own work carried out by him under our supervision and guidance. The matter embodied in this thesis has not been submitted in part or full to any other University or Institute for the award of any degree.

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NOMENCLATURE

a_i, b_i, c_i	Operating cost coefficients of i-th thermal power generating plant equation
P_{si}	active thermal power generation of i-th plant
$F_i(.)$	cost of thermal generation at ith plant
n	number of thermal plants
β	Total number of plants comprising of both thermal and hydro-units
$E(x)=\bar{x}$	expected value of random variable X
var(x)	variance of random variable X
C_{vPsi}	Coefficient of variation of random variable P_{si}
Cov(X, Y)	covariance of random variables X and Y.
P_D	System load demand
P_L	System transmission loss
B_{ij}, B_{uv}	transmission loss formula coefficients
$\underline{B}$	loss formula coefficient matrix
$\underline{P_s}$	Vector comprising of thermal generations
$\alpha(.)$	Lagrangian function
$\bar{F}_T(.)$	expected value of combined cost consisting of expected system power production costs and expected penalty costs
p	penalty cost
λ	Lagrange multiplier, transformation parameter
ε	error in the representation of the incremental operating cost
U	control vector
P_H	active power generated by a hydro-plant
h	basic head of a hydro-plant

Q	water discharge through the turbine
c	waterhead correction factor to account for variation in head with storage
ρ_j	water discharge needed through the turbine of j-th hydroplant to run the turboalternator at no-load.
m	superscript identifying m-th subinterval
J	water inflow to the reservoir of the hydroplant
X	water storage of a hydroplant at the beginning of a sub-interval
M	total number of subintervals
ρ	correlation coefficient
τ	the number of elementary periods which elapse before the water released at the upstream reservoir reaches the downstream
G	exact Hessian of a function
H	approximating Hessian of G
g	gradient of a function
t	small increment Δx

S Y N O P S I S

Optimal power (load) scheduling is an important problem for any electric utility system. Its importance has been further enhanced with the development of large integrated power systems. The solution of optimal power scheduling problem gives generation schedules at various generating plants, such that power demand is met at minimum possible operating cost and the system constraints are satisfied. The problem has assumed increasingly higher importance and priority as the operating costs continue to escalate and the main thrust of the system analysts being on the conservation of conventional fuels such as coal and oil. For these reasons, the optimal power scheduling problem still continues to be an important subject for carrying out extensive research for finding newer algorithms and methods for achieving the above mentioned objectives.

Early work in this area was that of Kirchmayer in fiftees who solved an all-thermal system problem, employing coordination equations derived by the Lagrange multiplier technique. In this formulation, the effects of the equality and inequality constraints imposed by the transmission and generating systems were either approximated or completely ignored. Carpentier in 1962 for the first time advanced an exact formulation of an all-thermal system dispatch problem, taking into consideration the several equality and inequality constraints of the system, which were

not considered by the earlier researchers. Subsequently, several algorithms for solution were proposed in the literature to solve the problem as formulated by Carpentier.

Despite the extensive research focussing on optimal power scheduling, most of the efforts to-date have been mainly centred around development of deterministic models applicable to steady state conditions. The system data has normally been assumed to be known with complete certainty i.e. deterministic. However, in practice the power demand and water inflow at various hydro-plants do vary randomly with time. In addition to this, there are several inaccuracies and uncertainties in the input-output characteristics of thermal units that may lead to deviations from the optimal operation. Thus it is essential to find out as to what extent the economy is influenced by these deviations. It is the intent of this research to develop and refine methodologies which can be applied to optimal power scheduling procedures while recognising stochastic nature of phenomenon.

A more rational approach to the optimal power scheduling problem has been attempted by incorporating quantitative representation of the inaccuracies as uncertainties in terms of probability and statistics followed by a quantitative evaluation of the influence of these uncertainties in generation schedules.

The objectives of investigations carried out in this thesis are: i) generalise the existing optimal scheduling procedures in order to take into account statistical uncertainties in the input data for optimal power scheduling problems and ii) evaluate the potential economic significance of optimal power scheduling considering statistical uncertainties. More specifically, the aim is to schedule the power output of each generator considering stochastic formulation of the input-output cost curves in a practical way with procedure of mathematical programming and minimise the expected value of fuel costs within the recognised constraints. The introduction of randomness in the optimal power scheduling calculations cannot reduce uncertainty, but it can lead to more realistic solutions in the system analysis. Therefore, stochastic analysis provides a valuable aid in power system planning and operation.

The thesis briefly reviews the past work in the area of deterministic optimal power dispatch problem and lays down the objectives and motivations for this research by identifying several inaccuracies and uncertainties which lead to deviations from deterministic optimal power schedules. A stochastic model of the objective function for the optimal power scheduling problem of a thermal system is formulated giving quantitative representation of inaccuracies and uncertainties in terms of probability and statistics. The stochastic model is reduced to its deterministic equivalent and the optimisation problem solved by an efficient algorithm, which is a direct extension of existing methods of analysis. An example demonstrates the suitability of this method for application to actual power systems. A suitable interpretation is assigned to the possible deviations in the value of costs associated with the two sets of schedules.

Optimal scheduling of hydrothermal system is tackled next. A long range stochastic hydrothermal scheduling model is formulated. The problem variables are treated as stochastic rather than deterministic. The optimisation problem is solved using an efficient decomposition technique. Intervalwise decomposition is carried out so as to reduce the complexity of the problem. In each sub-interval, hydro sub-problem is solved separately using conjugate gradient method and thermal subproblem using conventional λ-dispatch. A numerical example of 2-hydro and 2-thermal plants is solved and the results presented.

Generality of the stochastic formulation is next illustrated by applying it to solve an optimal scheduling problem consisting of a conventional as well as cascaded hydro-plants.

Variable metric minimisation algorithms require the determination of step size accurately in each iteration such that maximum reduction in function value is achieved in each iteration. Accurate minimisation along each search direction can be expensive in function evaluations and their gradients. Much more work recently has been devoted to the development of algorithms when accurate line searches are dropped in favour of stability requirement: that function must decrease at every step. Fletcher's modified variable metric method requires approximate step size and makes use of convex class of updating approximate inverse Hessian matrix. In Chapter 5, this method has been applied for the first time to the best of the author's knowledge to solve the problem of hydrothermal scheduling.

Finally, the main results of the thesis are reviewed and future lines of research are indicated.

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CHAPTER - 1

INTRODUCTION

1.1 INTRODUCTION AND MOTIVATION

Minimisation of the costs of generating electric energy has long been a concern of electric utilities. Optimal power dispatch problem allocates output among a group of generating units in a manner which minimises costs while satisfying a given set of constraints. Because of the large, complex and integrated nature of a modern electric power system, contemporary optimisation procedures use computers to make accurate analysis. With the continuing increase in the prices of the fuel and ever increasing emphasis on energy conservation, the need for more accurate analysis is emphasized. In real world situation, the criterion of perfection is never met and there are deviations between the model and the reality. This research increases the accuracy of the presently available optimal dispatch procedures by incorporating the effects of uncertain (stochastic) input data into the computer algorithm for analysis. This additional information permits optimal dispatch procedures to account for the random nature of the uncertainties which unavoidably affect the input data and which can cause significant changes in the solution of the economic dispatch problem.

1.2 STATE OF THE ART OF THE PROBLEM

Optimisation procedures for attaining the desired solution to the constrained optimal dispatch problem are divided into two general categories, viz. (i) approximate model, and (ii) rigorous model. Each of the two categories are differentiated by the type of variables manipulated. Aporoximate models utilise REDUCED SET of variables in which only the control variables appear in the mathematical program. The effects of network are modelled by mathematical functions involving only the controlled variables. Classical economic dispatch problem is formulated using reduced set of variables. Rigorous models utilise a FULL SET of variables consisting of all voltage magnitudes, phase angles, bus P and Q loads and P and Q generations. Methods using a full set of variables are termed as optimal power flow dispatch. Such methods require complete power flow solutions. The area of economic operation of electric power systems is explored fully from a tutorial point of view in [2-7, 147].

1.2.1 Approximate Optimal Dispatch Model:

All thermal system dispatch ignoring losses:

When the electric energy losses are small as in compact small networks, then the dispatching problem appears in its simplest form. The dispatch problem is to minimise fuel costs while serving the total load, and with all units operating between their minimum and maximum output limits.

$$\text{Minimise:} \quad \sum_{\substack{\text{all operating}\\ \text{units}}} \text{FUEL COST} \tag{1.1}$$

$$\text{subject to:} \quad \sum_{\substack{\text{all operating}\\ \text{units}}} \text{POWER} = \text{LOAD} \tag{1.2}$$

PMIN $\leq$ POWER $\leq$ PMAX for all operating units (1.3)

POWER = 0 for all non-operating units (1.4)

The FUEL COST for each power generating unit is a function of the POWER output.

As early as 1933 it was recognised that generating units should be operated at equal incremental cost [1]. The electric utility industry refers to this incremental cost as the system "Lambda" acknowledging its origin in the proof using Lagrange multipliers. A digital computer determines the dispatch by iterating on lambda using the incremental cost equations. Once the outputs of the generators add to the total load, the iterations stop.

Equal incremental dispatching is discussed in detail by Kirchmayer [2] and is also available in several other books such as Stevenson [3], Knight [4], El-Hawary and Christensen [5] and Nagrath and Kothari [6].

With the development of interconnected power systems and interconnections between electric utilities for the purpose of economic interchange of power, it became necessary to consider the transmission line losses in the system for achieving better economy.

Dispatching with transmission losses duly considered:

Transmission of electric power is always accompanied by energy losses. The largest loss component is heating computed by the current-square times the resistance. The greater the distance the higher the resistance, and the greater the power the higher the current at a given voltage. Therefore, when the generating units and loads are dispersed in a network, it is more economical to generate power at slightly higher incremental cost closer to the loads and incur lower losses, than to generate at the same incremental cost at a distant location.

The transmission losses enter into the constraint eqn. (1.2) as follows:

$$\sum_{\text{All units}} \text{POWER} = \text{LOAD} + \text{LOSSES} \qquad (1.5)$$

where the LOSSES are a non-linear function of the POWER variables. The practical method for handling this nonlinear constraint has been to fit a quadratic function of the POWER variables to the LOSSES function and derive penalty factors to modify each unit's incremental cost before equating it to the system lambda. It was first proposed in 1943 but became practical only after Gabriel Kron developed a series of loss invariant transformations, making it possible to compute the loss formula coefficients, termed as "B" constants [2]. The inclusion of transmission line losses for the optimal operation led to the incremental transmission loss formula [2,8] which when incorporated with incremental fuel cost led to the classical "Coordination equation method" [2-12] for optimal power dispatch. The implementation of loss formula dispatches showed a saving of about 1,50,000 dollars on a system with loads varying from 800 to 1900 MW and served by eight thermal plants [2]. The loss formula approach has gained wide popularity and is still being widely used by utilities [9,13,14] but does require some important judgements and approximations [15,16]. Recently, Aoki et al [17] have proposed many new algorithms using parametric quadratic programming, modified parametric quadratic programming and recursive quadratic programming for the solution of classic economic power dispatch problem.

1.2.2 Rigorous Optimal Dispatch Model

Overcoming the loss formula approximations require solving power flow equations representing steady state model of a power system while minimising the fuel costs. The power flow equations are obtained by equating the power injected into each node (by sources and sinks) with the power removed from the node (by network elements) and recognising that under steady state conditions the power system network can be represented by an admittance matrix. The single eqn. (1.5) is replaced by two SLFE (steady state Load Flow Equations) (1.6) and (1.7) for every node/bus in the network.

$$(\text{INJECTED REAL POWER})_{\text{NODE/Bus}} = (\text{REAL POWER GENERATED})_{\text{NODE/BUS}} - (\text{REAL LOAD CONNECTED})_{\text{NODE/BUS}} = \sum_{\substack{\text{All lines connected}\\ \text{to the node/bus}}} \text{LINE FLOWS} \qquad (1.6)$$

where LINE FLOWS are the real power flows away from the node/bus.

Eq. (1.7) implies that the reactive power summation around each node must equal zero.

$$\left(\begin{array}{c}\text{INJECTED BUS}\\ \text{REACTIVE POWER}\end{array}\right) = \left(\begin{array}{c}\text{GENERATED}\\ \text{VARS}\end{array}\right) - \left(\begin{array}{c}\text{LOAD}\\ \text{VARS}\end{array}\right) = \sum_{\substack{\text{All lines}\\ \text{connected to}\\ \text{the bus}}} \text{LINE VARS} \quad (1.7)$$

where GENERATED VARS are the reactive power generated at the bus, LOAD VARS are the reactive power demanded at the bus, and LINE VARS are the reactive power flows away from the bus.

In an oridinary load flow study [18], the power output of all generators (except one) are specified as well as all generator voltages, generator VAR limits, load WATTS and VARS are also specified. The load flow program based on any well known LF (Load Flow) methods such as Gauss-Seidel, Newton-Raphson, Fast Decoupled Load Flow etc. solves eqns. (1.6) and (1.7) to determine the unknowns such as generator VARS, bus voltages, angles, and all line flows alongwith system loss. The load flow is one of the earliest developments in the application of digital computer for power system studies and undoubtedly the most valued of all programs available to a power system analyst. An optimal power flow for optimising real power generation only, solves power flow eqns. (1.6) and (1.7) just like an ordinary load flow, with the exception being that generator powers are not specified, and instead min-max ranges for each generator power are given. The optimal load flow, using the fuel cost curves of the units, determines the optimal allocation of generation and thus the economic power output of the units through non-linear programming techniques, since the power flow eqns. (1.6) and (1.7) are inherently non-linear in nature.

Optimal power flow (OPF) has been the subject of continuous intensive research and algorithmic improvements continue since its introduction in the early sixties. The 1951 work of Kuhn and Tucker [19] provides the ground work for most of the optimal dispatch theory associated with power systems. Carpentier [20] is generally accredited with the first to formulate rigorously an exact optimisation problem using NLP techniques taking into consideration the system equality and inequality constraints. The problem is based on Lagrange-Kuhn Tucker formulation. The solution of the necessary conditions as derived from the Lagrangian gives the optimum strategy for economic load dispatch. The method is, however, quite slow and had convergence problems in the general case. It now only keeps an historical interest.

For finding the optimal power flow solution for optimal power dispatch, the actual constrained problem has been solved by using unconstrained minimization techniques like those of optimum gradient, conjugate gradient and Fletcher-Powell. These techniques are used for minimisation of cost after reducing the constrained problem to an unconstrained one with the help of Zangwill, Powell or any other standard transformation technique. The minimization techniques based on gradient methods obtain the minimum of an objective function by using the first and/or the second order derivatives. The 1968 work of Dommel and Tinney [21] presented a practical computer method for solving the OPF problem. There are various important contributions of this work such as a step by step procedure for implementing the optimal solution method on the computer, the expression of the Kuhn-Tucker conditions in terms of power system parameters, the equations for various sensitivity matrices, and a computer flow chart. The method of Dommel and Tinney uses a first order gradient technique. Except that it suffers from the necessity of choosing an appropriate step size, the method is very efficient since the computational procedure is constructed around the very fast Bonnville Power Administration's Newton-Raphson load flow program incorporating sparsity technique, optimal ordering scheme, compact storage scheme and therefore requiring less computer storage even for large systems. Among the other methods using gradient information, mention can be made of the efforts by researchers such as Abadie [22], Peschon [23], Alsac and Stott [24] and Padmore [25]. These methods vary in their generality, and in their treatment of inequalities on the dependent variables, which are accounted for through penalty functions [21, 24] through generalised reduced gradient method [22,23], and through the gradient projection method of Rosen [25,26].

The important methods for the economic dispatch problem based on second derivative information are the works of Sasson [27], Reid and Hosdorf [28], Nicholson and Sterling [29], and Rashed and Kelly [30]. In addition, Dommel and Tinney [21] have also reported experiences with the use of the reduced Hessian in various approximate forms. The above methods vary primarily in their treatment of inequality and equality constraints, which is done either implicitly through penalty factors [27,30] and explicitly through projection techniques [30] or quadratic programming [28,29]. These algorithms have in common the fact that the search direction at each iteration is obtained from the second derivative information. The definition of the Hessian matrix and its dimensions, however, vary depending on which variables are defined as controllable. In [27], both equality and inequality constraints are added to cost through penalty functions, thereby eliminating the need to differentiate between controllable and dependent variables. Essentially, all state variables are independent and controllable, and the dimension of the Hessian is then defined by the dimension of the state vector. In [28] and [29] the general simplex-based quadratic programming methods of Wolfe [28,31] and Beale [29, 32] are used to explicitly account for the inequality and equality constraints and to minimise the cost. In [30] an approximation to the reduced Hessian is explicitly computed with respect to the controllable variables which are defined as the voltage magnitudes and phase angles at the generation buses. The cost is then reduced through a Hessian search procedure. Equality constraints are handled via the Lagrange multiplier approach, whilst inequalities are accounted for by a projection method for controllable variables, and via penalty functions for dependent variables. In [33,34] advantage is taken of the sparse structure of a power system. Rather than updating the full inverse of Hessian at every step, the sparse triangular factors of the Hessian matrix are updated at every stage of the optimisation process. Only the non-zero elements of the triangular factors are updated and this procedure results in a more computationally efficient method.

Linear programming (LP) formulations have also been applied. An excellent survey on formulation and solution methods for the linearised problems is given in [35]. These formulations rely on the linearisation of both the constraints and the objective function. Some examples of these methods are found in Stott and Marinho [36], Lugtu [37], and Chan and Schweppe [38]. These approaches vary in the generality of the problem formulation and to some degree in the type of LP algorithm used. A dual LP formulation is given by Shen and Laughton [39] who obtained a fast solution for the problem of an all-thermal system under inequality constraints. This was a contribution to the on-line dispatching problem.

The literature does not seem to be unanimous in its preference for any of the three main methodologies described above, and much less for any particular technique. Many factors enter into a comparison of various approaches. In general, gradient methods are susceptible to slow convergence if the Hessian eigenvalues are widely spaced which is the case when the incremental generation costs have widely varying slopes [40,41]. Their advantages lie in the relatively simple computation of the gradient and their good compatibility with easily available Newton load flow algorithms. Second derivative based methods can be very efficient, but the computation of the Hessian is subject to some drawbacks. If the constraints are handled implicitly via penalty functions, the Hessian is sparse, but of higher dimension than necessary. Furthermore, the adjustment of the weighting coefficients of the penalty factors when a constraint is violated, is a non-systematic step which may lead to numerical difficulties [27, 42] and suboptimal solutions. If on the other hand the reduced Hessian is exactly computed with respect to the controllable variables only (by explicitly eliminating some of the variables through equality constraints), its computation is in general very involved and the resulting matrix is non-sparse. The Simplex based quadratic programming approaches used in [28,29] avoid the explicit computation of the Hessian but are computationally very demanding for large systems [29].

Linear programming algorithms have gained increased popularity in the area of real-time control of power systems because of their characteristics namely reliability, speed and adequate accuracy and scope for most engineering problems.

It is obvious that a great deal of research effort has gone into the solution of OPF problem. Much of the effort has been devoted to the improvement of convergence, the reduction of computation time and computer storage requirements. Algorithm improvements continue to emerge to deal with large system requirements. A two-stage procedure proposed by Wu et al [43] exploits the structural properties of the OPF problem while incorporating some features of the generalised reduced gradient method. Many recent proposed solution algorithms [44-49] decompose the problem into the real and reactive subproblems by exploiting the decoupling characteristics between the network voltage and phase angle. The real power optimisation (P-problem) is to minimise the production cost under the assumption that system voltages are held constant, and the reactive power optimisation (Q-problem) is to minimise the transmission loss under the assumption that real power generation is held constant. Due to loose coupling between two problems, the sequential optimisation of these provide a considerable advantage over the simultaneous optimisation of all control variables.

More recently, practical algorithms for energy control centre implementation have been developed and reported by Sun et al [50] and Burchett et al [51]. The results of a detailed study assessing the benefits to the Ontario Hydro system due to OPF implementation is reported by El-Kady et al [52].

1.3 APPROXIMATE VS RIGOROUS MODEL

Frequent recalculation of a new dispatch (2 minutes to 2 seconds) and limited computer size require that the applied method be simple in structure and fast in calculation [53]. The classical economic load dispatch method has provided a simple and fast solution procedure, making it an attractive technique for both real time dispatching and study programs. The disadvantages of approximate model are the need for generating updated B-matrix as system configuration or load level changes and the inability to include functional constraints. The inherent advantage of the rigorous model is the simplicity with which voltage, and reactive power as well as real power constraints can be formulated. An additional major benefit realised by the rigorous methods is that an ordinary load flow is executed as a byproduct, in addition to the dispatch, which closely resembles the actual system conditions, a security benefit is thus obtained. Since the load flow equations must be solved iteratively in rigorous methods, the disadvantages to these methods lie in the requirements of large computer memory and the amount of computation per iteration to reach a solution.

Happ [10] has conducted a comparison study to determine the difference in savings obtained by using the classical and rigorous dispatch procedures. He has shown that no significant difference in production costs is realised using either of these models. Happ in his excellent review paper [9] has reported the "Directions of Industry Concerning Economic Load Dispatch" in which it is stated that, 'The reason for employing more advanced techniques cannot be on the basis of savings alone, but because more rigorous models are required for executing different functions associated with the security of operations'. The security constrained economic load dispatch algorithms are presented in [36,37,54].

However, since the capacity of transmission lines is designed with sufficient margin, limits on transmission lines are not needed for the normal operating state. In the case of emergency states such as circuit outages of transmission lines, transmission line capacity limits will be treated in the emergency control to maintain the safety of the system. Since the chief aim of the optimal power dispatch is to operate the system economically, it is assumed in this thesis that classical economic load dispatch formulation is employed, and the economic operation and the emergency control may be treated separately.

1.4 HYDRO THERMAL COORDINATION

Falling water is valuable source of energy, but when it is limited in quantity its use for power generation is coordinated with thermal plant operation to minimise costs. The procedure for integrating the operation of hydro and thermal generations in a hydrothermal system for minimum cost of thermal generation has been referred to as hydrothermal coordination or scheduling. The coordination of hydroplants with the thermal plants requires the study of an year to days of operation in order to use the water to minimise costs. The dispatch problem now becomes time dependent, and includes the limits on the hydroplant energy output.

$$\text{Minimize} \sum_{\text{Time}} \sum_{\text{All Thermal units}} \text{FUEL COST} \tag{1.8}$$

$$\text{subject to:} \sum_{\text{All units}} \text{POWER} = \text{LOAD for each time period} \tag{1.9}$$

$$\sum_{\text{Time}} \sum_{\text{All hydro-units}} \text{ENERGY} \leq \text{ENERGYMAX for each set of hydro-units.} \tag{1.10}$$

where ENERGY is the integral of POWER over time. Equations (1.8) and (1.9) replace (1.1) and (1.2), which do not involve time. The total energy generated by the hydrosystem is limited to the maximum energy for the period as given by eqn. (1.10). The solution to the hydrothermal economic dispatch problem consists of determination of a plan for withdrawl of water for hydropower generation and the corresponding thermal generations so as to minimise total cost of thermal generations over a specified period while satisfying all the equality and inequality constraints imposed on the system.

The hydro-thermal coordination has been classified as long range and shortrange scheduling problems. The long range problem considers the yearly cyclic nature of reservoir water inflows and seasonal load demand and correspondingly a scheduling period of one year is used. The solution of the long range problem considers the dynamics of head variations through water flow continuity equation. The water inflows and load demand are expressed deterministically i.e. assumed to be known with complete certainty. On the other hand, the load demand on the power system exhibits cyclic variation over a day and a week and correspondingly scheduling interval for short range problem is either a day or a week. As the scheduling interval of short range problem is small, the solution of the short range problem can assume the head to be fairly constant. The amount of water to be utilised for short range scheduling problem is known from the solution of the long range scheduling problem.

Several researchers have tried to investigate the complex problem of hydrothermal coordination. An extensive, bibliography is presented in [55, 71]. The number of research papers in this field is so large that a critical review of all of them is beyond the scope of this thesis. A good overview with an extensive bibliography is presented in [56,57]. Only a brief presentation of some of the investigations is given here.

Various optimisation techniques have been used in the past for hydro-thermal scheduling. Among these are the variational calculus principles, the method of dynamic programming and the Pontryagin's maximum principle. In 1940, Ricard [58] obtained a set operating schedules for a hydrothermal system with no losses. His work was continued in 1953 by Chandler et al [59] who included transmission losses but with constant hydraulic head. The latter method was improved in 1958 by Glimn and Kirchmayer [60] who included variable head plants. They also reported the work of Kron, who developed equivalent equations. A set of scheduling equations was developed by Cypser [61]. These were developed under the assumption that variations in elevation and plant efficiencies can be neglected. Carey [62] suggested an approach that would linearise Cypser's equations. Watchorn [63] gives a set of equations to be satisfied in order to achieve maximum economy.

The above mentioned investigations employed the Euler equation of the calculus of variations. The work of Arismunander and Noakes [64] dealt with short range optimisation of hydro-thermal systems. All the necessary and sufficient conditions for optimality of the variational calculus were employed. In 1962, Drake et al [65] presented a dispatch formula based on the calculus of variations. This formula is restricted to the case where all the hydroplants operated with constant head. The system considered has series plants, multiple chains of plants, and intermediate reservoirs. Kirchmayer and Ringlee [66] presented a dispatch formula for a hydrothermal power system in 1964. Head variations were considered. The formula applies for power systems having one hydroplant. Discussing the work of Drake et al [65], Watchorn [63] points out the importance of considering variable head for the optimisation of such systems. In separate discussions of the same work Watchorn and Arismunandar point out that a river time delay of a couple of hours is highly important for accurate optimisation of many power systems.

Using dynamic programming Bernholtz and Graham [67,68] and Singh and Aggarwal [69] have shown how to obtain the optimal instantaneous operating policy of a hydrothermal power system. Dynamic programming was successfully applied to dispatching of two hydroplants in series on the Susquehanna river [70], but variational methods were considered to be more appropriate for large hydro-systems [66]. Successive approximations of dynamic programming are applied in three different cases and compared in a two hydro-unit system [72]. Rees and Larson [73] describe a dynamic programming application to the scheduling of a pumped storage hydro unit, two hydro plants, sixteen thermal plants, and three interchange contracts overtime spans from two days to one year. The algorithms based on Dynamic Programming as pointed by Dahlin and Shen [74] suffer from a 'Curse of Dimensionality'. Moreover, the computational time required for the solution of a realistic size problem becomes prohibitive. In 1970, however, Arvanitidis and Rosing [75,76] have shown how dynamic programming can handle networks of several hydroplants located on the same river. Their idea is to convert the water stored at each plant into its at-site and down stream generating capability and to sum over all plants. The result is a one-dam network which receives, stores and releases potential energy. However, it should be pointed out that although this technique is a useful approximation, it does not always describe the system adequately, in the sense that local constraints are ignored. For example, if the distribution of the water reserve between the reservoirs is uneven, spillage may occur when it is not predicted by the aggregated model. So once the optimal operation of the aggregated reservoir has been found, there remains the difficult combinatorial problem of distributing the total reserve of potential energy among the real reservoirs.

Calculus of variations is useful for the constrained power systems. Computer solutions for constrained power systems may be obtained for the special case where the operating region for all power sources in the system forms a convex set. This is not possible when, for instance, some of the power sources have a piecewise continuous or discrete type of operating characteristics [74,77] One method that has been successfully applied to constrained systems of several hydro plants is the Maximum Principle of Pontryagin [77-81].

In fact, Dahlin and Shen [77]have shown that this method applies equally well to systems with unbounded, bounded, continuous, piecewise continuous and discrete operating characteristics.

Dahlin and Shen's results, however, are only valid for systems with run-of-river plants. If reservoirs were included, inequality constraints of the type $G(x) \leq 0$ would be required, and these are difficult to incorporate in a Maximum Principle solution. Thus they did not consider systems with reservoirs.

Narita [78] did not ignore the reservoirs. In fact, his paper is the first serious attempt to solve the long range scheduling problem with the Pontryagin's Maximum Principle. Its main drawback is that it solves a discrete problem with a continuous optimization technique, when the discrete version is easy to apply and is much more efficient [81]. Dillon and Morsztyn [80] have extended Narita's model in an interesting way. They have, for instance, included restrictions on the power transitions along the transmission lines so as to take into account the system stability requirements. They have also carried out the optimisation with respect to both active and reactive power simultaneously.

Narita's problem has also been solved with the Discrete Maximum Principle to yield excellent results [81]. In fact, Oh [81] claims that the use of discrete version of the Maximum Principle (DMP) instead of the continuous version results in a considerable saving of computer time even when applied to continuous problems. In addition, Oh states the Discrete Maximum Principle is much more effective than

Dynamic Programming because the number of time and state variables does not have to be increased in order to improve the accuracy of the solutions. Nagrath, Dayal, and Kothari [82] have also used DMP for solving hydrothermal scheduling problem with transmission losses duly considered.

Various nonlinear programming techniques have also been applied to the solution of hydrothermal scheduling problem. Sokappa [83] has formulated the long range problem as a non-linear programming problem. The gradient of the cost function was evaluated and the steepest descent method used to obtain the solution of the problem, starting from a known initial schedule. Such a procedure becomes impossible in view of large dimensional requirements for problems of multireservoir hydro-thermal problems.

Agarwal and Nagrath [84] presented a new algorithm based on conjugate gradient technique for the solution of multi-storage hydro and multi-thermal system. Lagrangian multipliers take care of equality constraints while inequality constraints are considered using Powell's penalty function in the formulation. This formulation uses decomposition technique to conserve memory requirements of a large system. Hicks et al [85] have applied the same technique for the solution of a large scale problem of Bonneville Power Administration System with 38 reservoirs and 45 run-of-river plants. A study of 49 time periods produced a dispatch with nearly 1 percent more energy output than a cut-and-try solution. Saha and Khaparde [86] have used the method of feasible directions for the problem solution. Their method ensures the satisfaction of active constraints while determining the direction of the move towards the optimum. Augmentation of the objective function is automatically avoided. The earliest work on optimal load flow for hydrothermal systems is probably due to Ramamoorthy and Rao [87] . Head variations are neglected and the problem is solved using NLP techniques. The discretisation process makes the problem one of large dimension. However, a method of splitting the problem into one of smaller dimension is proposed. The main difficulty with their approach is handling of some of the constraints imposed by the hydro subsystem. The minimum norm formulation of a hydrothermal optimal load flow (HTOPF) with isolated hydroplants is given by El-Hawary et al in[88] . In a recent paper [89] , El-Hawary et al discuss the formulation of HTOPF and derive the necessary conditions for optimality of the problem. Newton's method has been applied to solve for the optimum operating strategies. However, the method is computationally very slow.

More recently, Vemuri and Hill [90] presented a sensitivity analysis algorithm based on the successive sweep method. The feasibility of using the algorithm is demonstrated for a system with two thermal plants and two hydroplants. Other methods used include the method of local variations [91] , and the principle of Progressive Optimality [92].

Wan et al [93] presented an approach based on the marginal cost method for problem formulation. It is demonstrated that compared to dynamic programming approach, this method required much less memory and one-sixth of computation time. Shaw and Bertskas [94] formulated the problem as large nonlinear mixed integer optimisation problem. Execution time of the algorithm was found to be linear in number of thermal units and roughly quadratic in number of coupled units.

1.5 LITERATURE SURVEY OF STOCHASTIC METHODS USED IN SOLVING POWER SYSTEM PROBLEMS

Consideration of the electric power systems as a network characterised by random variables has been studied to various degrees for many years. Smith and Benner [95,96] are a few examples of an early attempt to consider uncertainty associated in the power system. Generation failures and line outages were tabulated. The tabular results formed the basis for evaluating system reliability. The methods employ a statistical equilibrium approach whereby, in a large group of networks, the probability of loss is related to the probability of restoration. Benner determined both single and multiple time outage hours per year.

Various other segments of power system analysis have partially included the effects of uncertainty. Power flow calculations were treated as functions of random variables for the first time by Borkowska [97] . In this approach, various well known probability densities including Gaussian and binomial were employed to model the load demand at various busses. Independence among real and reactive components of the loads were assumed. A linearised approximation to the load flow solution in conjunction with convolution techniques were utilised to determine solutions. A more recent analysis of the stochastic load problem was made by Sauer [98]. A major advantage of this method is that independence of random variables need not be assumed. The Gram-Charlier Series was used to obtain p.d.f.'s for the line loading thus allowing for a more accurate representation. The overall result of Sauer's approach is a more accurate analysis of a realistic load flow problems. The main drawback of the method appears to be the storage requirements associated with retaining many terms of the multivariate Gram-Charlier series [99].

During the same period as Sauer's work, the problem of power system stability under uncertainty was analysed by Burchett [100]. The method relies on the calculation of the joint probability that all eigenvalues (of the linearised system) lie in the left hand plane. The works of Burchett and Sauer were similar in the sense that both incorporated the multivariate Gram-Charlier series as a model for the general p.d.f. Both were successful in utilising the series approximaton for a practical example.

The area of multivariable probabilistic analysis has been one of the increasing interest during recent years. The survey paper of Heydt [101] provides a good overview of current topics in the field of stochastic analysis of power systems. Time series analysis techniques [102,103] are sometimes considered for use in modelling time varying processes such as power system demands. A recent work by Manichaikul and Schweppe [104] illustrates the importance of random variables in power system analysis. Manichaikul and Schweppe consider the problem of determining a stochastic model for electric power demands based on physical considerations. Two basic physical aspects, "the stochastic aspects of the use of individual pieces of equipment and the product flow and storage structure of the individual processes", are considered. Power demands are modeled by a two state Markov process [105]. A model proposed in reference [104] is utilised to represent product flows and storage. The proposed technique is tested utilising data from several industrial consumers.

1.6 OUTLINE OF THE THESIS

The central theme of the work is the introduction of statistical uncertainty in the optimal power dispatch algorithms in order to obtain realistic solutions to the complex problem of load (power) scheduling in power systems. A brief outline of the material discussed in the following chapters is given below.

Chapter 2 deals with load scheduling of an all-thermal system. The chapter describes several inaccuracies and uncertainties that lead to deviations from the optimal operation. A critical survey of the earlier work incorporating the effects of these inaccuracies and uncertainties in the economic dispatch procedure is then given. The deterministic version of classical economic power dispatch problem is formulated and the solution strategy outlined. A stochastic model of the objective function representing fuel costs is formulated, giving quantitative representation of naccuracies and uncertainties in terms of probability and statistics. The optimisation problem is solved by reducing the stochastic model to its deterministic equivalent. The new method is illustrated by first applying it to a simple 2-plant sample system neglecting transmission losses. The versatality of the method is then illustrated by applying it on a system [2] with loads varying from 800 to 1900 MW and served by eight thermal plants. The costs associated with generation schedules obtained from the proposed method are compared with costs associated with generation schedules obtained from classical economic thermal power dispatch. A suitable interpretation is assigned to the possible deviations in value of the costs associated with the two sets of schedules. The Chapter is appended with a possible method of Monte Carlo simulation to obtain sample estimate of the expected value and variance of generator output variable via incremental production cost model incorporating random coefficients.

Chapter 3 addresses itself to the stochastic approach to the problem of scheduling a hydrothermal system. The chapter commences with critical review of existing stochastic formulations of the problem. A long range stochastic hydrothermal scheduling model is then formulated considering statistical uncertainties in hydroreservoir water inflows, system load demand, and thermal power generations because of inaccuracies in a thermal plant operating cost curve. The problem variables are treated as stochastic rather than deterministic.

The possible deviations in the values of the stochastic variables from their respective expected values are incorporated as penalties in the objective function. The optimisation problem is then formulated as minimisation of the expected value of the sum of fuel costs and penalty costs subject to the system equality and inequality constraints. The optimisation problem is solved by an efficient decomposition technique. Intervalwise decomposition is carried out so as to reduce the complexity of the problem. Each subproblem is separately solved by using conjugate gradient method. A numerical example of a power system consisting of two hydro and two thermal plants is solved and the results are presented.

Chapter 4 shows the generality of the stochastic formulation of Chapter 3 by applying it to solve a optimal scheduling problem consisting of a conventional as well as cascaded hydro-plants. Numerical example illustrates the proposed method.

In Chapter 5 Fletcher's modified variable metric method of 1970 has been applied for the first time to the problem of hydrothermal scheduling. The minimisation algorithms require the determination of step size accurately in each iteration such that maximum reduction in function value is achieved during the iteration. Fletcher's modified method requires approximate step size and makes use of convex class of updating the approximate inverse Hessian matrix whereby the optimal solution is always assured and computational time greatly reduced. An algorithm based on Lagrangian formulation for optimal hydrothermal scheduling is given. Intervalwise decomposition has been carried out so as to reduce the complexity of the problem. Each subproblem is separately solved by using Fletcher's variable metric method. The potential of the algorithm presented has been fully demonstrated through a sample system study.

The sixth chapter gives overall conclusions of the work reported in this thesis alongwith few suggestions or possibilities for further research in this important area.

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C H A P T E R - 2

THEORETICAL DEVELOPMENT, FORMULATION AND SOLUTION OF THE STOCHASTIC OPTIMAL THERMAL POWER DISPATCH PROBLEM

2.1 INTRODUCTION

The economic thermal power dispatch problem calculates least costly loadings of the generating units. It is, therefore, quite natural that it has attracted a great deal of research efforts during the last three decades. The input to the optimal power dispatch calculation consists of parameters describing the individual units and the transmission network, the output provides commands for changes in governor settings. When the input information is inaccurate or commands are not followed exactly, a loss of performance occurs.

Despite extensive research focussing on optimal thermal power dispatch problem, much of the effort to date has involved the development of deterministic models applicable to steady state conditions. Most of these attempts assume the system data to be deterministic. It means that all input information is known with full certainty and that the optimal plans of dispatch are always realised exactly. In practice,it is not so. Looking at the process of operation planning Fig.2.1 one can see that several inaccuracies and uncertainties lead to deviations from optimal operation. These are mainly on the one hand the inaccuracies and uncertainties in the input information which is needed by the operation planning procedure and on the other hand the modeling and solution inaccuracies

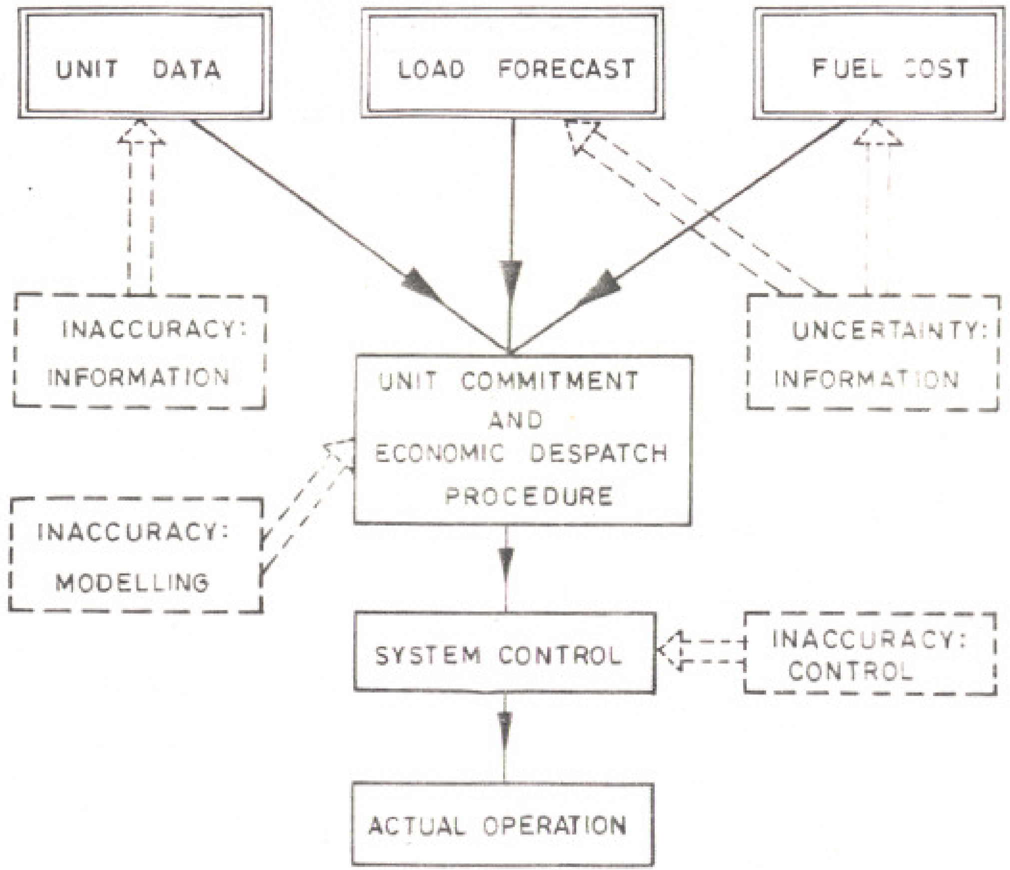

Fig. 2·1 Optimal power system operation: Inaccuracies and uncertainties.

in the unit commitment and economic dispatch procedures. The effects of the inaccuracies in general result in an increase in the overall cost [90]. From this situation the question arises as to what extent the economy is influenced by these deviations. It has only been in comparatively recent times that researchers have been successful in modelling elements of the non-deterministic nature of optimal thermal power dispatch problem. It is the intent of the present research to develop and refine methodologies which can be applied to optimal thermal power dispatch while recognising stochastic nature of the data. The value of the stochastic model lies in the fact that due consideration can be given to uncertainties. The results obtained provide a useful guide to decision making concerning the plant operation, which is a judicious combination of engineering and management judgements.

In this chapter first the basic model of the deterministic optimal thermal power dispatch problem is described and then the sources of variability in the operating cost functions of the problem are listed. The limited existing work in this area is critically reviewed. Stochastic optimal thermal power dispatch problem is then formulated and solved by direct extension of the existing methods of analysis. The method is illustrated by applying it to a simple lossless 2-plant system. The possible economic significance of the method is illustrated by carrying out studies on a large sample system. In these methods a prior knowledge of thermal generating unit commitment has been assumed.

2.2 BASIC MODEL

In optimal load dispatch problem, generating operating cost characteristic is the most important factor. The major component of generating operating cost for fossil plants is the cost of fuel input per hour, while the cost of maintenance, water etc. contribute only a negligibly small portion [2]. The operating characteristics of fossil plants can be expressed in terms of million calories/h, or directly in terms of Rs./h versus output in megawatts. If P_{si} is the real power output of the ith generating unit, $I_i(P_{si})$ is the heat input of that unit and γ the fuel price, then the operating cost of the plant is

$$F_i(P_{si}) = \gamma \cdot I_i(P_{si}) + K_i(P_{si}) \tag{2.1}$$

where $K_i(P_{si})$ is the additional costs for maintenance, supplies and water etc.

Since it is difficult to express the costs of labour, maintenance, supplies etc. as a function of the output, it is usual to assume these costs as a fixed percentage of the fuel cost at that output.

The power output of the plant is increased by sequentially opening a set of valves at the inlet to the steam turbine. The throttling losses in a valve are large when it is just opened and small when it is fully opened. As a result, the operating cost of a plant has the form shown in Fig.2.2. For dispatching purposes this operating cost per unit for generator 'i' is usually approximated by quadratic polynomial [2].

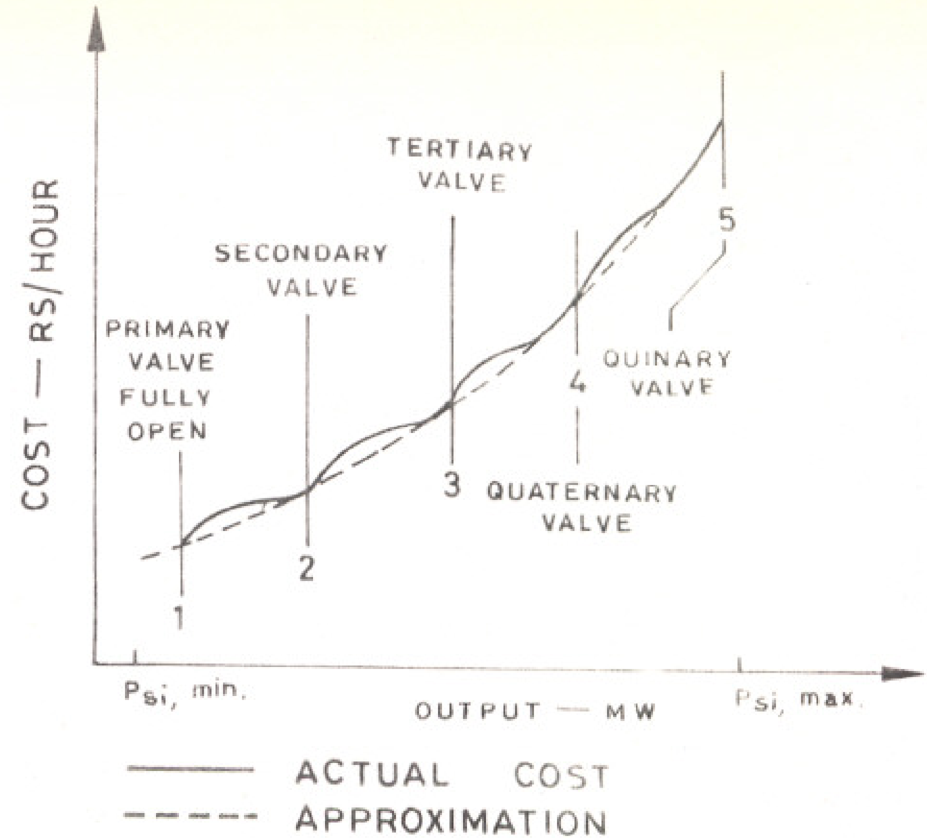

Fig. 2·2 Operating cost of a fossil fired generator.

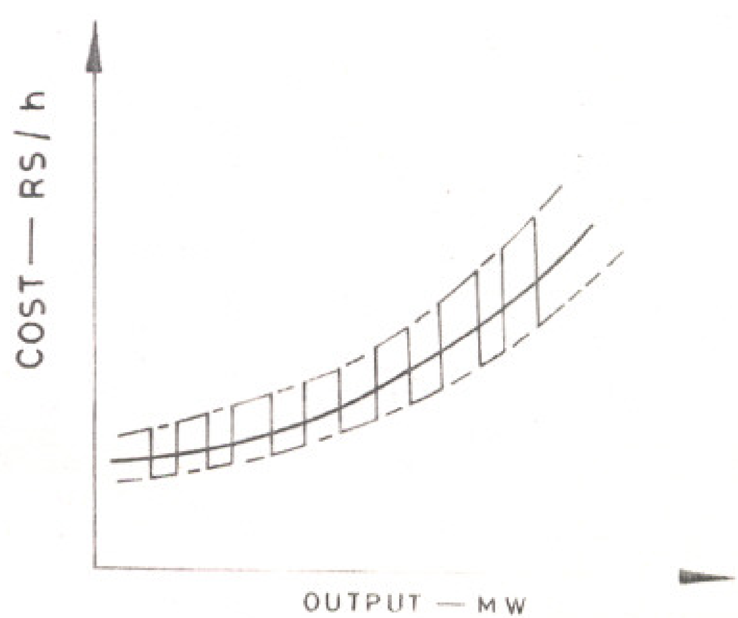

Fig. 2·3 Stochastic operating cost as a function of power generated.

$$F_i(P_{si}) = (a_i P_{si}^2 + b_i P_{si} + c_i) \quad \text{Rs/h} \tag{2.2}$$

where a_i, b_i, c_i are positive constants.

The above approximation is presently used by many utilities. It is believed to be a good approximation, considering the limited data on units cost performance.

2.3 CHARACTERISATION OF CLASSICAL OPTIMAL DISPATCH PROBLEM

Classical economic dispatch problem is formulated using the vector of controlled real power generations $\underline{P_s}$ and a $\underline{B}$ matrix equation to represent the system losses as a function of $\underline{P_s}$.

$$\text{Minimise } F(\underline{P_s}) = \sum_{i=1}^{n} F_i(P_{si}) \tag{2.3}$$

where n is the total number of thermal generating units.

$$\text{Subject to: } \sum_{i=1}^{n} P_{si} = P_D + P_L \tag{2.4}$$

$$P_{si,min} \leq P_{si} \leq P_{si,max} \qquad i=1,\ldots,n \tag{2.5}$$

Here transmission loss $P_L = \underline{P_s}^T \, \underline{B} \, \underline{P_s}$ (2.6)

$F_i(P_{si})$ in eqn.(2.3) is given by eqn.(2.2)

2.4 SOURCES OF VARIABILITY

The operating cost functions representing the performance characteristics of thermal plants are computed by calculation of overall thermodynamic performance of a unit consisting of boiler, turbine, condenser, heat cycle and associated plant auxiliaries. Such cost functions are inaccurate in most cases. The inaccuracies are deviations from input information given on the data sheets from their values during actual operation. They are caused due to the following reasons [106] :

- inaccuracies in the process of measuring the basic data used for computation of thermodynamic performance of the unit.
- deviations from the computed thermodynamic performance of the unit because of errors encountered in operation due to operating at other than standard pressure and temperature.
- effect of time on equipment conditions which influences some of its operating characteristics, notably its efficiency.
- inaccuracies resulting from inability to hold generation at exact desired output.
- fuel cost variations
- load forecasting errors
- inaccuracies introduced by various types of transmission loss equations.

Further, because of the great difficulty of determining the dependency of maintenance costs from the power output P_{si}, the addition costs for maintenance, supplies and water are very inaccurate.

All these factors taken together can mean inaccuracies of great magnitude in the steady-state operation. As a consequence, instead of having a single operating cost curve representing the bogey performance of the unit what really exists is a narrow band such as shown in Fig. 2.3 in which the real curve would lie. Operating cost function is, therefore, known only as a statistical distribution.

From such data it is not possible to get a deterministic minimum but only that such a minimum obtained from the calculation performed has a statistical distribution.

The deviation in the operating cost characteristics results in a new optimum schedule and hence a new optimum cost, $\hat{F}$. The original optimum generation schedule is no longer an optimum one and using this set of schedules one gets an operating cost, F for the period considered. This results in the loss of economy, ΔF, given by

$$\Delta F = (\hat{F} - F) \qquad \text{Rs/h} \qquad (2.7).$$

2.5 REVIEW OF LITERATURE

The accuracy considerations in economics of power systems with only thermal plants were first studied by Glimn et al [106]. Appendix A presents an analytical determination of the loss of economy resulting from an incremental cost displacement multiplier $(1+\varepsilon)$ for one of two identical units on the same bus and $(1-\varepsilon)$ for the other unit. The following conclusions are drawn for the two source system investigated:

(a) The loss in hourly economy varies as the square of the per unit error ε in the representation of the incremental operating cost.

(b) For a given value of ε, the loss in hourly economy varies directly as the square of the incremental-cost level and inversely as the average slope of the incremental-cost characteristics of the sources.

The authors also studied the effect of error from inability to hold generation at exact desired output. The output of one source was displaced by $+\Delta P_s$ MW and the output of another source by $-\Delta P_s$ MW from the desired economic values. It was found that the loss of hourly economy varies as the square of the ΔP_s MW deviation from the optimum schedule. The main limitation of this analysis is the consideration of two identical units with no transmission losses. A sensitivity study of the optimal dispatch was reported in Ref. [107]. The paper analyses the sensitivity of the total operating cost with respect to unit cost functions, transmission loss formula, load variations and control error. The information and control errors are translated into several classes of mathematical variations. The contribution of this paper is a set of simple expressions which provide good estimates of the cost penalties with various types of errors. The benefits of any proposed improvements can be evaluated by simply subtracting cost penalty associated with system having that improvement from the cost penalty obtained without the improvement. Approximate calculations, based on sample data supplied by AEP show that cost penalties of the order of one percent of the total operating cost may result from (a) errors in the unit cost functions, (b) absence of the transmission losses from the dispatch formulation, and (c) unit operating limit changes. An investigation of the cost penalties associated with load variations and errors in the derivatives of the transmission losses show that the total operating cost is rather insensitive to these factors.

Russian engineers M.Keel et al [108] discuss the problem of optimal scheduling of thermal power systems with due considerations of various risks. In such situations, the thermal generation powers, P_{si}, in the cost functions are treated as random variables [109]. This implies that optimal generation schedules are realised on the average. The deviations, ΔP_{si}, of random variables from their expected values $\overline{P}_{si}$ are treated as stationary stochastic processes at fixed $\overline{P}_{si}$. The stochastic objective function is reduced to its deterministic equivalent in tabular form. The authors employ numerical differentiation for the determination of incremental cost rates. The paper, however, does not elaborate the type of stochastic processes used. Nevertheless, the authors claim that the consideration of random factors enables to give economy of fuel.

Edwin et al in their paper [110] analyse the extent to which the economy of short term operation of thermal power systems is influenced by (a) the approximation in the operation planning method and (b) by the inaccuracies of the steady-state operation costs, start up costs and load forecast. In order to reduce the computation time and storage requirements, the operation planning problem is decomposed into unit commitment and loading which are then treated sequentially. The operation planning methods compared are (a) Method I-heuristic method using priority lists for committing units ("merit order") followed by loading of the committed units by the principle of equal incremental costs and (b) Method II-investigated is the one using the mathematical optimisation technique of dynamic programming using the penalty search process.

The above two operation planning methods are compared on the basis of 12 thermal power systems of different structures using two typical load curves for each system. The number of units per system varied from 6 to 24. The authors considered the stochastic nature of the inaccuracies in all types of input data using the Monte Carlo simulation technique. In the first step, which has to be repeated several times, inaccurate input data are simulated by taking random values from given density functions around the exact value of each input information by means of a random number generator. With this set of simulated inaccurate input data the schedules are determined, the operating costs of which are calculated with the accurate input data which represent the unit performance during actual operation. Thus during each simulation the operation costs are determined which are dependent on the stochastic combination of the inaccuracies. The difference in the cost increase caused by the inaccuracies of the input data is then calculated. The set of the n cost differences of the n simulation runs is analysed statistically. The inaccuracies are assumed to be normally distributed. The investigations which were carried out by simulation for the 12 systems reveal that average cost increases were 1.59 percent for operation planning method I and 1.56 percent for operation planning method II. Thus the operation costs were shown to be very much dependent on the accuracy of the input data.

The second influence investigated was the inaccuracy of start up cost functions. As these costs are only about one percent of the total operation costs the effects of these inaccuracies are very small. Yet if start up costs were neglected completely, operation costs would increase upto one percent. Finally, the inaccuracy of the load forecast was varied and again the inaccuracies of the remaining input data were held at their medium values. The economy of operation is influenced by this inaccuracy more than by those of the other two types.

Method I and method II were also compared under the assumption that all input data are accurate. Operation cost decreases by 0.3 percent on average by means of more sophisticated method of dynamic programming using penalty search procedure when compared with method I. In most systems the cost decrease was not affected by inaccurate input data. Therefore, it is economically clearly justified to employ more sophisticated unit commitment methods wherever possible.

Viviani et al in an interesting paper [111] successfully applied probabilistic method to optimal power flow problem. The proposed procedure is essentially an optimal power flow followed by an evaluation of the statistical properties of the control vector U, given the statistics of uncertain parameters such as load demand. The uncertainties are accounted for by treating load demand P_D at different buses as random variables. The deviations, ΔP_D, of random variables from their expected values $\bar{P}_D$ are treated as stationary stochastic processes. The algorithm minimises a quadratic cost function to find optimal value of control vector U^* subject to the random fluctuation of P_D. These random fluctuations in P_D are approximately linearly related to the control vector U^* allowing the p.d.f. of U to be calculted from the p.d.f. of ΔP_D. The stochastic optimal dispatch solution, therefore, results in a p.d.f. representation of the changes in the optimal control vector ΔU^* about an operating point U^*. The algorithm employs the multivariate Gram-Charlier series to statistically model the p.d.f. of the control vector in terms of the statistical moments. The p.d.f. of ΔU^* provides useful information about the likelihood of certain events associated with U^*. The probability of events pertaining to U^* are valuable in assessing the ability of a system to perform at minimal cost. The algorithm proposed is primarily a planning tool rather than an operating procedure since computation time, while not prohibitive, is not fast enough to be on-line.

2.6 CHARACTERISATION OF THE STOCHASTIC OPTIMAL THERMAL POWER DISPATCH PROBLEM

A more rational approach to optimal dispatch is quantitative representation of the inaccuracies as uncertainties in terms of probability and statistics followed by a quantitative evaluation of the influence of these uncertainties on generation schedules.

The objectives of this investigation are:

(a) generalise the existing load dispatch procedures in order to take into account statistical uncertainty in the system operating cost; and

(b) evaluate the potential economic significance of optimal power dispatch considering statistical uncertainty. More specifically, the aim is to schedule the power output of each generator considering stochastic cost curves in a practical way using mathematical programming techniques and minimise the expected operation costs within recognised constraints. It may be noted that the introduction of randomness in the economic dispatch calculations cannot reduce uncertainty, it can lead to more precise statements about uncertainty and, hence to more realistic economy in system operation.

2.6.1 Stochastic Modeling:

Stochastic models evolve through two main routes. Some stochastic models can be derived by directly employing probabilistic concepts to small sub-divisions of the system. An alternative method of obtaining stochastic model is to take a deterministic model and transform it into a stochastic model by:

- introducing random variables as inputs or as coefficients or both
- introducing equational errors as disturbances.

Neither route of approach is more "correct" than the other, because both types of models are approximations in any case. What is important in each approach is to make the randomness reflect a more real world situation.

2.6.2 Formulation:

A stochastic model of function F_i, is formulated by taking the otherwise deterministic cost coefficients a_i, b_i and c_i as random variables. A specific way of reducing a stochastic model to its deterministic equivalent one is to take its expected value. Therefore, the expected operation cost is given by

$$\bar{F} = E[\sum_i (a_i P_{si}^2 + b_i P_{si} + c_i)] \quad (2.8)$$

$$\bar{F} = \sum_i (\bar{a}_i P_{si}^2 + \bar{b}_i P_{si} + \bar{c}_i) \quad (2.9)$$

The deterministic equivalent set up of the objective function as given by eqn. (2.9) in which all the random variables have been replaced by their expected values is ineffective. Therefore, it is necessary to pose the problem differently.

It is possible to use different objective functions in risk taking situations [108, 109]. In risk taking situations it is also necessary to take into account the random deviations of values of controllable variables from their expected values. It means, that controllable variables, P_{si}, must be regarded as random variables. Any possible deviation of operation cost coefficients and load demand from their respective expected values is manipulated through the randomness of generator power P_{si}. The randomness of P_{si} implies that power balance eqn.(2.4) is not a rigid constraint to be satisfied. Whatever the proposed solution will be, only deficit or surplus probabilities of generated power can be estimated. The random variables are assumed to be normally distributed and statistically independent. In practice these density functions have to be established with reasonable approximation. In the case of normal distribution, the expected value must be characterised by a standard deviation obtained from engineer's experience.

The expected operation cost is given by

$$\bar{F} = E\left[\sum_i (a_i P_{si}^2 + b_i P_{si} + c_i)\right] \tag{2.10}$$

$$= \sum_i (\bar{a}_i \bar{P}_{si}^2 + \bar{b}_i P_{si} + \bar{c}_i + \bar{a}_i \text{ var } P_{si})$$

$$= \sum (\bar{a}_i \bar{P}_{si}^2 + \bar{b}_i P_{si} + \bar{c}_i + \bar{a}_i C_{vPsi}^2 \bar{P}_{si}^2) \tag{2.11}$$

where C_{vPsi} is the coefficient of variation of random variable P_{si}. It is the ratio of standard deviation to the mean and is a measure of relative dispersion or uncertainty in the random variable. A coefficient of variation of zero implies no randomness, or in other words complete certainty about the value of the random variable.

2.6.3 Representation of System Losses

The economic importance of representing transmission losses in scheduling of generation has been well established and this can be most conveniently achieved by the use of loss formulae [2]. The transmission loss being function of random variables P_{si} is itself random. The B-matrix can be advantageously used to evaluate the expected transmission loss. With P_{si} as statistically independent, the expected value of transmission loss as derived in Appendix-C is given by

$$\bar{P}_L = E\,[P_L]$$

$$= E\,[\sum_i \sum_j P_{si}\, B_{ij}\, P_{sj}] \qquad (2.12)$$

$$= \sum_i \sum_j \bar{P}_{si}\, B_{ij}\, \bar{P}_{sj} + \sum_i B_{ii}\, \text{var}\, P_{si} \qquad (2.13)$$

The variance of transmission loss has been neglected for ease of analysis. This is a reasonably justified assumption to the extent that even in the deterministic case the B-matrix represents but only approximate transmission loss which is never more than 5 percent and usually less than 2 percent of the total power transferred from generators to the using substations.

2.7 DETERMINISTIC EQUIVALENT

One classical approach in stochastic programming [112] for the random resources problem is to transfer the expected value associated with deficit or surplus of random constraints to the objective function, with appropriate penalty factors. Therefore, the expected objective function $\bar{F}_T$ representing the combined system cost is formulated as the sum of (i) expected operating costs, and (ii) expected cost of deviations, a penalty term proportional to the expectation of the square of unsatisfied load because of possible variance associated with random variable P_{si}. This penalty term is given by

$$p.E\left[\left(\bar{P}_D+\bar{P}_L-\sum_i P_{si}\right)^2\right] \tag{2.14}$$

Therefore

$$\bar{F}_T = E\left[\sum_i \left(a_i P_{si}^2+b_i P_{si}+c_i\right)\right] + p.E\left[\left(\bar{P}_D+\bar{P}_L-\sum_i P_{si}\right)^2\right]$$

$$=\left[\sum_i \left(\bar{a}_i \bar{P}_{si}^2+\bar{b}_i \bar{P}_{si}+\bar{c}_i+\bar{a}_i C^2_{vPsi} \bar{P}_{si}^2\right)\right] + p.E \left[\left(\bar{P}_D+\bar{P}_L-\sum_i P_{si}\right)^2\right] \tag{2.15}$$

The optimisation problem is formulated as follows:

$$\text{Min } \bar{F}_T = \left[\sum_i \left(\bar{a}_i \bar{P}_{si}^2 + \bar{b}_i \bar{P}_{si}+\bar{c}_i+\bar{a}_i C^2_{v\,P_{si}} \bar{P}_{si}^2\right)+p.E\left\{\left(\bar{P}_D+\bar{P}_L - \sum_i P_{si}\right)^2\right\}\right] \tag{2.16}$$

$$\text{subject to} \quad \sum_i \bar{P}_{si} = \bar{P}_D + \bar{P}_L \tag{2.17a}$$

and

$$\bar{P}_{si,min} \leq \bar{P}_{si} \leq \bar{P}_{si,max} \qquad i=1,\ldots,n \tag{2.17b}$$

With equality constraint defined as in eqn. (2.17a), the objective function simplifies to:

$$\bar{F}_T =\left[\sum_i \left(\bar{a}_i \bar{P}_{si}^2+\bar{b}_i \bar{P}_{si}+\bar{c}_i+\bar{a}_i \bar{C}^2_{v\,P_{si}} \bar{P}_{si}^2\right) + p.E.\left\{\sum_i \left(\bar{P}_{si}-P_{si}\right)^2\right\}\right]$$

$$= [\sum_i (\bar{a}_i \bar{P}^2_{si} + \bar{b}_i \bar{P}_{si} + \bar{c}_i + \bar{a}_i \bar{C}^2_{vP_{si}} \bar{P}^2_{si}) + p\{ \sum_i (\mathrm{var}\, P_{si}) \}]$$

$$= \sum_i [\bar{a}_i \bar{P}^2_{si} + \bar{b}_i \bar{P}_{si} + \bar{c}_i + \bar{C}^2_{vP_{si}} (\bar{a}_i + p) \bar{P}^2_{si}] \qquad (2.18)$$

Thus, the problem simplifies to minimisation of $\bar{F}_T$ as in eqn. (2.18), subject to the constraints in eqn. (2.17).
The constrained minimisation problem is transformed to an unconstrained one by application of Kuhn-Tucker theory involving Lagrange multipliers [19] .

$$\alpha(\ldots) = \sum_{i \in s} [\bar{a}_i \bar{P}^2_{si} + \bar{b}_i \bar{P}_{si} + \bar{c}_i + C^2_{v P_{si}} (\bar{a}_i + p) \bar{P}^2_{si}]$$
$$- \lambda [\sum_i \bar{P}_{si} - \bar{P}_D - \sum_i \sum_j \bar{P}_{si} B_{ij} \bar{P}_{sj} - \sum_i B_{ii} C^2_{vP_{si}} \bar{P}^2_{si}]$$

Differentiating the Lagrangian function with respect to $\bar{P}_{si}$ leads to the following set of coordination equations:

$$(\bar{b}_i + 2\bar{a}_i \bar{P}_{si}) + \lambda \sum_j (2B_{ij} \bar{P}_{sj}) + \lambda 2 B_{ii} C^2_{vP_{si}} \bar{P}_{si} + 2(\bar{a}_i + p) C^2_{vP_{si}} \bar{P}_{si} = \lambda \qquad i = 1, \ldots, n \qquad (2.19)$$

The first two expressions in the set of coordination equation (2.19) are the incremental production cost and the incremental transmission cost similar to those in traditional coordination equations of conventional deterministic economic dispatch [2]. The expression

$$\lambda \cdot 2 B_{ii} C_{v P_{si}}^{2} \bar{P}_{si} + 2 (\bar{a}_i + p) C_{vP_{si}}^{2} \bar{P}_{si}$$

results in increasing the incremental cost of received power . This expression depends upon the fluctuations of P_{si} and is the risk premium in the risk taking situation posed in the economic dispatch formulation. The risk premium is also directly proportional to the amount of generator output P_{si}.

The scheduling of an n-plant system requires the solution of the set of n-simultaneous eqns. (2.19). For a more direct approach to the solution, eqn. (2.19) has been re-arranged as

$$\bar{P}_{si} = \frac{\left(1 - \frac{\bar{b}_i}{\lambda} - \sum_{j \neq i} (2B_{ij} \bar{P}_{sj})\right)}{\frac{(2\bar{a}_i + 2(\bar{a}_i + p) C_{vP_{si}}^{2}}{\lambda} + 2B_{ii}(1 + C_{vP_{si}}^{2})} \qquad (2.20)$$

Equation (2.20) for the net generation at a given plant is readily evaluated by the iterative form of solution [2].

2.8 APPLICATION OF THE METHOD

For the sake of analysis, the value of the penalty cost is assumed to be two times the value of 'a' coefficient in the operation cost curve equation of a generator.

To illustrate the new method, it will be first applied to a simple 2-plant sample system neglecting transmission losses. The plant incremental cost coefficients are 2

$\bar{b}_1 = 2.0,\quad \bar{a}_1 = 0.005,\quad \bar{b}_2 = 1.5\ ,\quad \bar{a}_2 = 0.005$

$p = 2\ \bar{a}_2 = 0.01$

Assuming that (1) both units are connected and operating, (2) minimum load of each unit is 20MW, and (3) maximum capability of each unit is 100 MW.

Using this data and the appropriate values of coefficient of variation in eqn. (2.20) the resulting expected generation schedules are shown in Fig. 2.4. The particular values of coefficient of variation used are also indicated therein. Consequently, expected schedules that are different from deterministic schedules are obtained when $C_{vP_{si}}$ is other than zero for any of the two generators. There is a shift in generation from one generator to the other depending upon the values of coefficient of variations.

The increase in production cost in dollars per hour for different schedules is shown in Fig. 2.5 as a function of the system load of the two sets of values of $C_{vP_{si}}$. The curves indicate an increase in cost of operation when both generators are operating at schedules other than their maximum or minimum limiting values. When either generator is operating at its limiting value, the deterministic and stochastic solutions are identical.

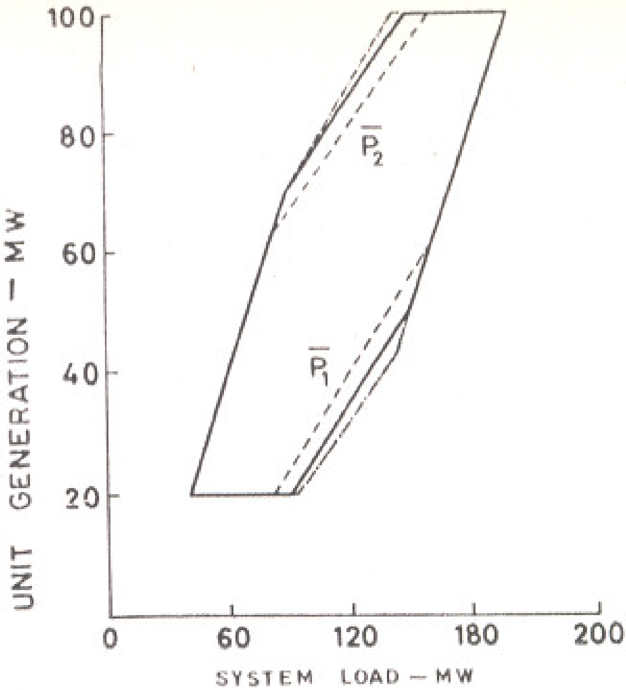

Fig. 2·4 Generation schedule for two – unit system.

——— $C_vP_1 = 0{\cdot}0$, $C_vP_2 = 0{\cdot}0$

- - - - - $C_vP_1 = 0{\cdot}0$, $C_vP_2 = 0{\cdot}2$

—·—·— $C_vP_1 = 0{\cdot}25$, $C_vP_2 = 0{\cdot}05$

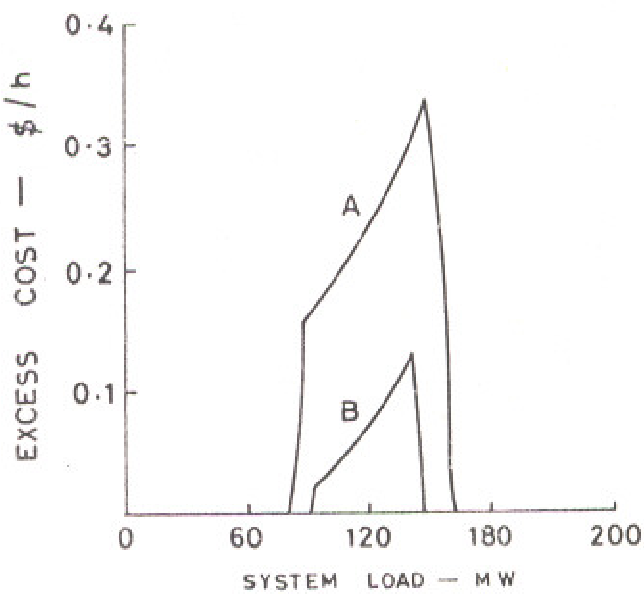

Fig. 2·5 Excess cost incurred in deviation deterministic schedules.

A — $C_vP_1 = 0{\cdot}0$, $C_vP_2 = 0{\cdot}2$

B — $C_vP_1 = 0{\cdot}25$, $C_vP_2 = 0{\cdot}05$

2.8.1 Studies on Large Sample System

A large system is now used to illustrate the method and to evaluate the possible economic significance of the method. The data [2,113] for the loss formula coefficients as well as the incremental production cost coefficients of a simplified representation of a system are given in Tables 2.1 and 2.2, respectively. The coefficients in Table 2.2 were valid for the range of loading presented in the study. The additional data assumed are the values of the penalty costs. For deterministic cases, all $C_{vP_{si}}$ are set equal to zero. In the two non-deterministic cases studied, the values of coefficient of variation are:

Case-1: $C_{vP_{s1}} = 0.10$, $C_{vP_{s2}} = 0.10$, $C_{vP_{s3}} = 0.10$, $C_{vP_{s4}} = 0.10$,
$C_{vP_{s5}} = 0.10$, $C_{vP_{s6}} = 0.10$, $C_{vP_{s7}} = 0.10$, $C_{vP_{s8}} = 0.10$,

Penalty cost $p = 0.0120$

Case-2: $C_{vP_{s1}} = 0.10$, $C_{vP_{s2}} = 0.10$, $C_{vP_{s3}} = 0.10$, $C_{vP_{s4}} = 0.10$,
$C_{vP_{s5}} = 0.10$, $C_{vP_{s6}} = 0.10$, $C_{vP_{s7}} = 0.10$, $C_{vP_{s8}} = 0.10$

Penalty cost $p = 0.0240$.

The non-zero values of the coefficient of variation studied in the above two cases are typically those obtained by the possible method of Monte Carlo simulation as described in Appendix-B.

Table 2.1: TRANSMISSION LOSS FORMULA COEFFICIENTS (B_{ij}x10^2)

i	j	B_{ij}	i	j	B_{ij}	i	j	B_{ij}
1	1	0.07863	1	7	-0.02892	4	5	0.00179
2	2	0.06098	1	8	-0.03292	4	6	-0.00707
3	3	0.09163	2	3	0.04624	4	7	-0.00876
4	4	0.02646	2	4	0.01246	4	8	-0.00992
5	5	0.02311	2	5	-0.01218	5	6	0.01224
6	6	0.03723	2	6	-0.01810	5	7	0.00721
7	7	0.06285	2	7	-0.01750	5	8	0.00378
8	8	0.12140	2	8	-0.01754	6	7	0.02166
1	2	-0.00999	3	4	0.01243	6	8	0.01682
1	3	-0.01402	3	5	-0.01198	7	8	0.05768
1	4	-0.00695	3	6	-0.02204			
1	5	-0.01136	3	7	-0.02530			
1	6	-0.02076	3	8	-0.02841			

Table 2.2: TABULATION OF PLANT DATA

Plant i	$\bar{a}_i$	$\bar{b}_i$
1	0.004100	1.280
2	0.002200	0.796
3	0.000950	1.809
4	0.002145	0.657
5	0.001110	0.889
6	0.006000	0.300
7	0.010400	0.635
8	0.006350	0.572

Fig. 2.6 gives the time varying system load used in studying the optimal dispatch of the sample system. The shape of the load curve is believed to be reasonably typical of that experienced over the high load portion of the day by an electric

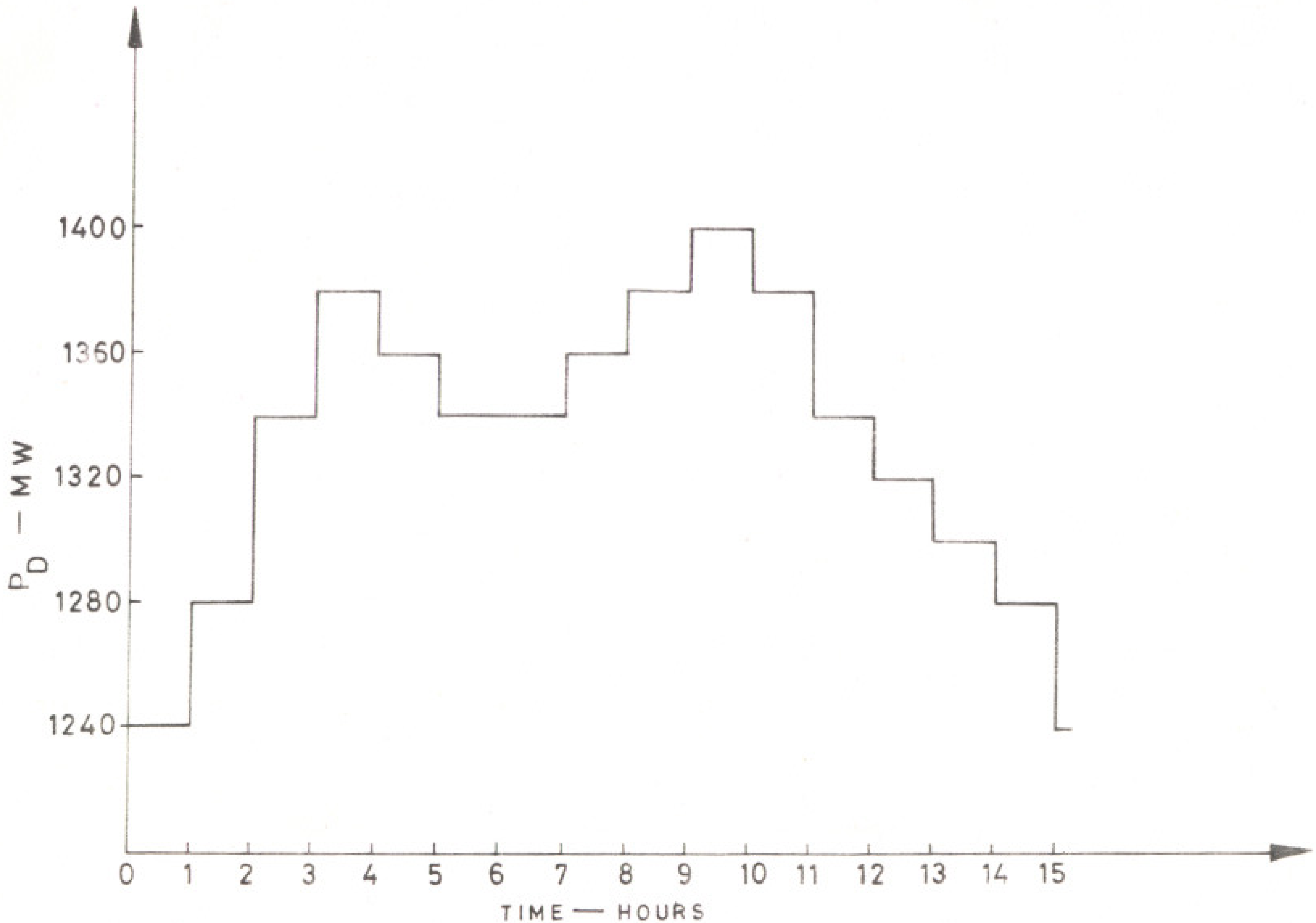

Fig. 2·6 Typical load — curve

utility system. Table 2.3 lists the various loads on the system and their duration in hours.

Table 2.3: SYSTEM LOAD DURATION

S.No.	Load (MW)	Duration (Hrs)
1	1240	1
2	1280	2
3	1300	1
4	1320	1
5	1340	4
6	1360	2
7	1380	3
8	1400	1

The deterministic generation schedules obtained from the solution of eqn. (2.20) for the zero values of coefficient of variation are given in Table 2.4.

The expected generation schedules obtained for the values of coefficient of variations and penalty costs in case I and case II are given in Tables 2.5 and 2.6, respectively.

Plants 2 and 5 remain at their maximum loads of 210MW and 310MW respectively and are not included in the Tables. There is a shift in generations as is evident from these Tables. This has a definite effect on the excess cost incurred in deviation from the deterministic schedules. Figure 2.7 shows the excess cost incurred in deviation from deterministic schedules.

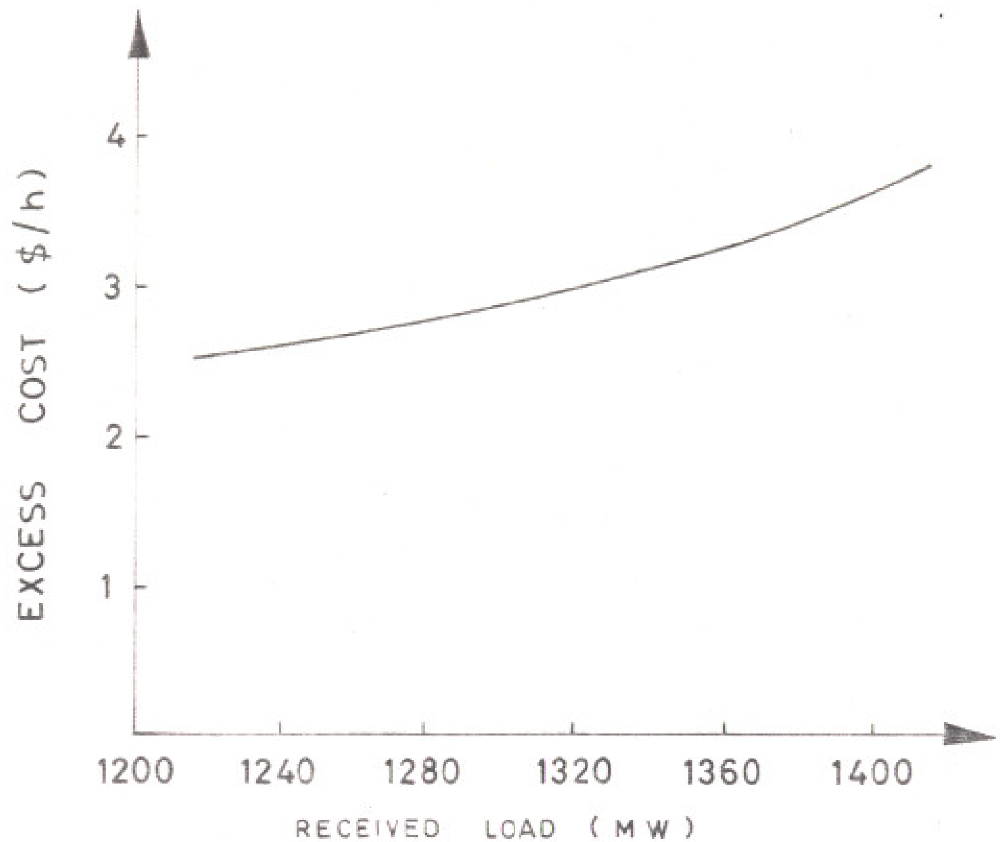

Fig. 2·7 Excess cost incurred in deviation from deterministic schedules.

Table 2.4: OPTIMAL DETERMINISTIC SCHEDULES

P_D (MW)↓ / P_{si} (MW)→	P_{s1}	P_{s3}	P_{s4}	P_{s6}	P_{s7}	P_{s8}
1240	137.20	61.05	272.76	145.83	65.25	101.17
1280	145.49	72.91	282.81	151.48	68.26	105.30
1300	149.67	78.80	287.87	154.34	69.78	107.39
1320	153.87	84.66	292.95	157.23	71.31	109.48
1340	158.10	90.46	298.05	160.15	72.85	111.57
1360	162.36	96.24	303.17	163.08	74.40	113.68
1380	166.64	101.99	308.30	166.05	75.96	115.79
1400	170.94	107.71	313.46	169.04	77.53	117.92

Table 2.5: STOCHASTIC OPTIMAL SCHEDULES
CASE-I C_{vP_s} =0.1 FOR ALL GENERATORS
PENALTY COST= 0.012

P_D (MW)↓ / P_{si} (MW)→	P_{s1}	P_{s3}	P_{s4}	P_{s6}	P_{s7}	P_{s8}
1240	138.18	66.43	266.06	145.65	66.13	101.46
1280	146.55	78.07	276.15	151.40	69.21	105.64
1300	150.77	83.84	281.22	154.31	70.76	107.75
1320	155.02	89.58	286.32	157.25	72.33	109.86
1340	159.29	95.28	291.43	160.21	73.90	111.98
1360	163.58	100.96	296.57	163.20	75.49	114.11
1380	167.90	106.60	301.72	166.21	77.09	116.24
1400	172.24	112.21	306.89	169.24	78.69	118.38

Table 2.6: STOCHASTIC OPTIMAL SCHEDULES
CASE-II C_{vp_s} =0.1 FOR ALL GENERATORS
PENALTY COSTS = 0.024

P_{si} (MW) / P_D (MW)	P_{s1}	P_{s3}	P_{s4}	P_{s6}	P_{s7}	P_{s8}
1240	138.93	69.68	260.39	145.70	66.91	101.86
1280	147.36	81.14	270.46	151.51	70.04	106.08
1300	151.61	86.82	275.52	154.45	71.62	108.20
1320	155.88	92.47	280.60	157.42	73.21	110.33
1340	160.18	98.08	285.70	160.41	74.81	112.47
1360	164.49	103.67	290.81	163.43	76.43	114.61
1380	168.84	109.22	295.95	166.47	78.05	116.76
1400	173.20	114.75	301.10	169.53	79.68	118.91

Savings (in dollars per hour) figures would be still higher by a factor 2 to 3 if the old data used in this sample study is updated by escalating the incremental production cost curves by a factor of 2 to 3.

Consideration of stochastic curves in the economic dispatch problem may significantly alter optimal generator outputs. Optimal values of generator outputs depend upon the values of coefficients of variation and the value of penalty cost 'p' and the penalty cost, in turn, depends upon the subjective decision of the power control engineer. More important, however, is an evaluation of the potential economic value of the consideration of stochastic curves in the economic dispatch. Such an evaluation is summarised in Table 2.7 where the cost of operating the sample system to supply the 15-hour load of Table 2.3 according to deterministic and stochastic cost curves is given. The combined no-load production costs for the 8 generators in the system are $1057 per hour. Savings attributable to stochastic curves would appear to be economically attractive if the weightage to penalty costs amount to more than about 2 percent of the total operating costs under conventional dispatch. This conclusion is based on the fact that the inclusion of the effect of transmission losses in the conventional economic dispatch is widely justified by savings of a few tenths of one percent in total fuel consumption [114] .

Table 2.7: COMPARISON OF DISPATCH CONSIDERING DETERMINISTIC AND STOCHASTIC PRODUCTION COST CURVES

Value of coefficients of variation	Penalty cost	Weightage to Penalty costs	Saving in total fuel cost (stochastic schedules)
	Values Expressed as Percentage of total Fuel cost (Deterministic Schedules)		
CASE-I $C_{vP_s}=0.1$			
for all generators	0.012	2.71	0.1044
CASE-II $C_{vP_s}=0.1$			
for all generators	0.024	2.98	0.1180
CASE-III $C_{vP_{s1}}=0.2$	0.012	1.11	0.1670
$C_{vP_{s2}}=0.17, C_{vP_{s3}}=0.13$	0.024	1.79	0.3230
All other values zero	0.048	2.74	0.7840

2.9 CONCLUSIONS

Conventional economic thermal power dispatch method allocates generation schedule to the individual thermal generating units based upon deterministic cost function ignoring inaccuracies and uncertainties. Such generation schedules will result in the lowest expected cost but this cost is also associated with a relatively large variance which can be interpreted as risk measure. Deterministic schedules may, therefore, render the solution non-optimal to a power control engineer who may like to avoid the risk element. The proposed method has the desirable features of incorporating inaccuracies and uncertainties in the economic dispatch procedure. The method proposes that to generate a robust economic dispatch procedure, the power control engineer should take account of a second best but deterministic approach which is to minimise simultaneously the expected cost and its sensitivity to deviations. Thus, an additional penalty cost is imposed. The proposed stochastic model results in an enhancement of the optimal value of the expected cost over the deterministic conventional dispatch model. The resulting increase in the value of expected cost has been interpreted as the risk premium which a power control engineer is expected to pay in making risk aversion decisions for accounting of the inaccuracies in the input data for optimal power dispatch problem. Results for a system consisting of eight thermal units have been presented using the new approach which has been found to be extremely simple and easy to implement.

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CHAPTER - 3

THEORETICAL DEVELOPMENT, FORMULATION AND SOLUTION OF THE STOCHASTIC OPTIMAL HYDROTHERMAL DISPATCH PROBLEM

3.1 INTRODUCTION

A hydrothermal generating system is composed of a thermal system and a hydroelectric system, linked to the load centers through the transmission lines, as shown in Fig. 3.1.

The objective of optimal system operation is to determine a generation schedule for each plant of the system which minimises the operating cost over a planning period. The operation costs include fuel costs for thermal units, purchase costs from neighbouring systems, and penalties for failure in load supply.

The availability of limited amounts of hydroelectric energy, makes the optimal operation problem very complex, because it creates a link between an operating decision in a given stage and the future consequences of the decision. That is, if we deplete the stocks of hydroelectric energy, and low inflow volumes occur, it may be necessary to use very expensive thermal generation in the future, or even fail to supply the load. On the other hand, if we keep the reservoir levels high through a more intensive use of thermal generation, and high inflow volumes occur, there may be spillage in the system

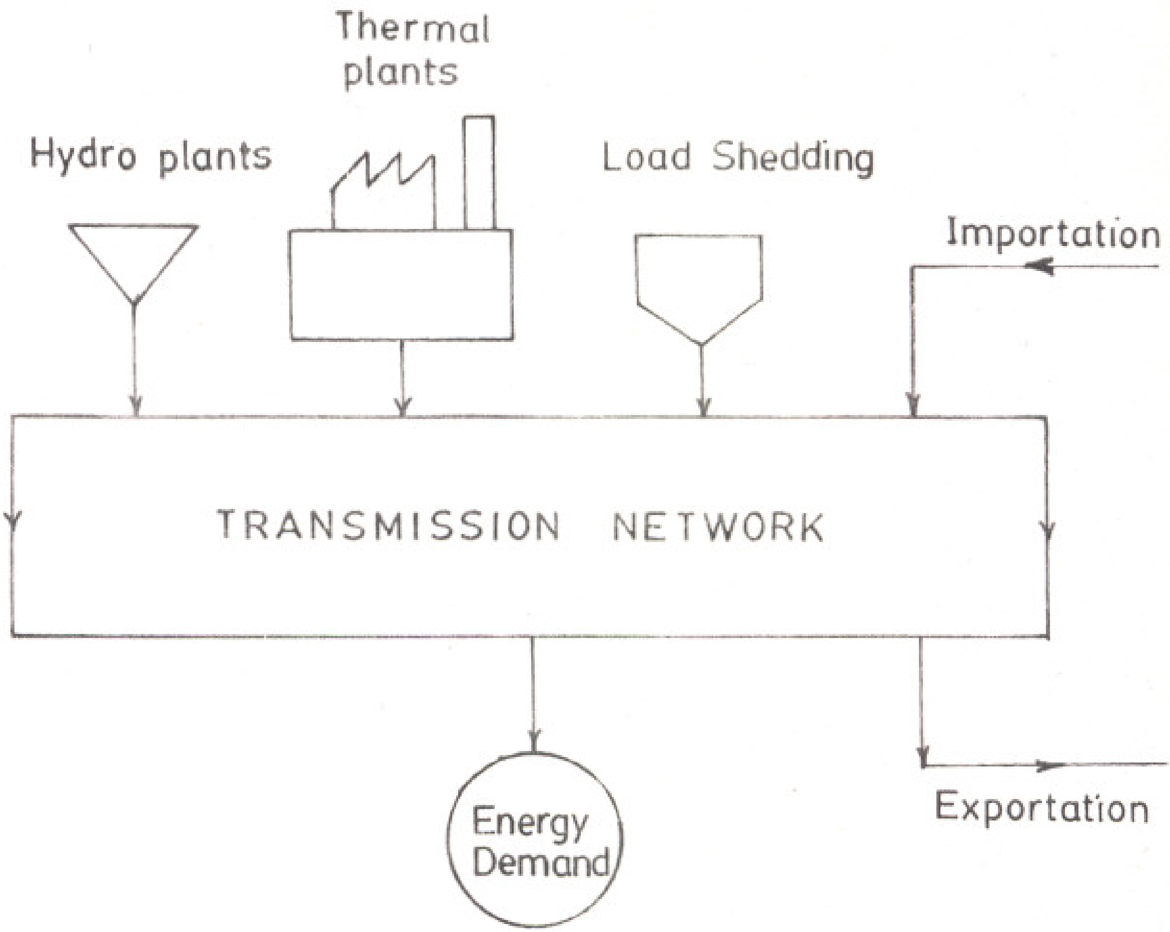

Fig 3.1. Schematic diagram of a hydrothermal system.

which means a waste of energy and consequently, higher operating costs. Therefore, hydrothermal scheduling is basically a multi-stage decision process. The existance of multiple interconnected reservoirs characterise the problem as large scale and complex also because (a) the objective function which comprises operating costs of thermal plants is nonlinear and, (b) the expression for the hydropower generation is nonlinear due to the product of outflow and head in the expression. Further, for a long range scheduling problem with a planning period of one year, it is impossible to have perfect forecasts of the future inflow sequences and in a certain measure, of the future load itself. Therefore, the operation problem is essentially stochastic as identified by the IEEE Group on Operating Economics [115].

This chapter proposes a novel approach to the hydrothermal scheduling problem that deals with stochastic nature of the data in a more general and precise way. The approach is to characterise the random nature of the data by random variables and analyse the problem by developing stochastic decision model. An equivalent deterministic problem based on the stochastic characterisation of the data is obtained. The principle of duality decomposition [123] is then applied to separate all the production units of the problem into thermal subproblem and hydro subproblem. The thermal subproblem is solved by classical dispatch. The hydro subproblem is optimised using the conjugate gradient method of Fletcher and Reeves [116].

3.2 HYDRO-PLANT PERFORMANCE MODEL

A basic physically based relationship between the active power generated (in MW) by a hydro-unit and the rate of water discharge, Q(in m^3/sec), and the effective head, h(in metres) is given by

$$P_H = 0.0085\, Q.h.\eta\cdot(Q,h) \tag{3.1}$$

The efficiency η is a function of both variables Q and h. Due to diversity of installation characteristics, numerous variants of model given by eqn. (3.1) exist. In the present work, model proposed by Narita et al [117] characterising variable head hydroplant performance is used. The corresponding expression for hydro generation at the j-th plant during the m-th sub-interval relating the water head, the storage of the reservoir at the beginning and end of the interval, and the discharge rate is given by a relation of the form

$$P^m_{Hj} = h_j \left[1 + \frac{c_j}{2}(X^m_j + X^{m+1}_j)\right](Q^m_j - \rho_j) \tag{3.2}$$

where c_j is a known constant.

The state equation representing the hydro system dynamics is

$$X^{m+1}_j = X^m{}_j + J^m_j - Q^m_j \tag{3.3}$$

Substituting eqn. (3.3) into eqn. (3.2) we get

$$P^m_{H_j} = h_j\left[1 + 0.5c_j(2X^m_j + J^m_j - Q^m_j)\right](Q^m_j - \rho_j) \tag{3.4}$$

3.3 CHARACTERISATION OF DETERMINISTIC LONG RANGE SCHEDULING PROBLEM

A power system consisting of 'n' thermal plants and $(p-n)$ hydro-plants is considered. A discrete-time representation of the continuous time optimal scheduling problem is presented here. The total interval of optimisation is divided into M equal subintervals, each subinterval being assumed to be of unit length for simplicity. The hydrostations are assumed to be operating with reservoirs which are independent of each other. In the deterministic formulation it is assumed that the load demand P_D^m, reservoir inflows J_j^m, $(m=1,\ldots,M)$, over the optimisation interval are known accurately enough in advance. The initial and final storages represented by the vector X_j^1 and X_j^{M+1} respectively are specified. These values represent the constraint on the amount of water to be consumed over the optimisation interval. That is,

$$X_j^{M+1} - X_j^1 - \sum_m J_j^m + \sum_m Q_j^m = 0, \quad j=1,\ldots,(p-n); \quad m=1,\ldots,M \tag{3.5}$$

represent the total water availability constraints.

The power balance equality constraint is

$$P_D^m = \sum_{i=1}^{n} P_{si} + \sum_{j=n+1}^{p} \tilde{P}_{H_j} - P_L^m \qquad m=1,\ldots,M \tag{3.6}$$

This constraint implies that the total power generated should be equal to the sum of total load demand and transmission losses in any subinterval m.

The upper and lower bounds on the rates of water discharge, water storage and thermal generations in each subinterval constitute the inequality constraints.

$$Q_{jmin} \leq Q_j^m \leq Q_{jmax} \qquad j=1,\ldots,\beta-n;\ m=1,\ldots,M \quad (3.7)$$

$$X_{jmin} \leq X_j^m \leq X_{jmax} \qquad j=1,\ldots,\beta-n;\ m=1,\ldots,M \quad (3.8)$$

$$P_{si,min} \leq P_{si}^m \leq P_{si,max} \qquad i=1,\ldots,n;\ m=1,\ldots,M \quad (3.9)$$

The optimisation problem is to determine the discharges Q_j^m, and the thermal generations P_{si}^m, such that the objective function, representing the total thermal power generation cost,

$$F = \sum_{m=1}^{M} \sum_{i=1}^{n} F_i(P_{si}^m) \qquad (3.10)$$

is minimised subject to the constraints of eqns. (3.3) to (3.9). The function $F_i(P_{si}^m)$ for the i-th thermal station is considered to be a quadratic of the form

$$(\bar{a}_i \, P_{si}^{m^2} + \bar{b}_i \, \bar{P}_{si}^m + \bar{c}_i) \qquad \text{Rs./h} \qquad (3.11)$$

In eqn. (3.6), P_L^m, transmission loss during the m-th subinterval, is expressed in terms of hydro and thermal generations [5,8] (listed in Appendix-C) as

$$P_L^m = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{si}^m B_{ij} P_{sj}^m + \sum_{u=n+1}^{\beta} \sum_{v=n+1}^{\beta} P_{Hu}^m B_{uv} P_{Hv}^m$$

$$+ 2 \sum_{j=1}^{n} \sum_{v=n+1}^{\beta} P_{sj}^m B_{jv} P_{Hv}^m \qquad (3.12)$$

The loss coefficients B_{ij} and B_{uv} are assumed to be available for the given power system.

Equations (3.3) to (3.12) constitute the deterministic model of the hydrothermal scheduling problem. Using these equations and starting with initial feasible values for water discharges, the optimisation problem can be solved efficiently. However, this model being inherently deterministic is inadequate to account for the random nature of the uncertainties which unavoidably affect the input data and which can cause significant changes in the solution of scheduling problem.

Next section briefly reviews a few of the papers available in the literature which treat some of the aspects of stochastic scheduling problem.

3.4 REVIEW OF LITERATURE ON STOCHASTIC HYDROTHERMAL SCHEDULING

A review of literature reveals that attempts to account for inaccuracies of the forecasted quantities such as system load demand and water inflow at various hydroplants were first applied to short range hydrothermal scheduling problem. These attempts consider frequent updating of the system predispatch obtained from the optimising method applied to the scheduling problem. Ringlee [118] has developed a water usage monitor for making on-line corrections for forecast error in load demand. Hara at el [119] presented a procedure for determining a first-order estimate of the change in the optimal schedule due to a small order parametric change in load or water inflow.

For the long range shceduling problem, uncertainty about river inflows and loads has been dealt with via two basic approaches:

- Methods based on stochastic dynamic programming
- Methods based on non-linear programming.

The list of references in [120-121] is rather comprehensive in this area. In the following a brief review of stochastic optimisation methods for multireservoir system is presented.

Aggregation of the multi reservoir and hydroplant system into a single equivalent reservoir and hydroplant model and solution by stochastic dynamic programming (SDP) is one of the earlier approaches that has been used [76,122, 124]. The main disadvantage of this approach is a computational one, since full SDP for systems with more than three or four reservoirs is computationally infeasible. Its main drawback is that local constraints on contents of reservoirs, water flows, and hydroplant generators are not accounted for. Moreover, disaggregation of the equivalent reservoir release policy and hydroplant generation among the actual reservoirs and hydroplants in the system may not be straightforward. The method does have usefulness, however, in a number of applications such as long range planning studies and can perform satisfactorily for systems where reservoir and inflow characteristics and behaviour are sufficiently "similar" to justify aggregation into a single reservoir and hydroplant model.

Application of SDP with successive approximations to a parallel multireservoir hydro system has been reported in [125-128]. The successive approximations involves a 'one-at-a-time' stochastic optimisation of each reservoir. The procedure is repeated over all the reservoirs until convergence is attained. It converges monotonically in a finite number of steps [125]. The SDP procedure yields "locally" feasible and optimal feedback reservoir release policies. These policies are not of the "global" feedback type and the solution is sub-optimal in a global sense. However, when the SDP successive approximations method is used to schedule the operation of the system in an "open-loop" mode, then the reservoir release and cost policies are expected to be close to optimal.

A limited number of papers have reported the application of non-linear programming to the problem of optimal stochastic hydrothermal scheduling. Agarwal [129] presents an efficient computational algorithm using non-linear programming approach with first-order gradient techniques for long range hydrothermal scheduling so as to minimise the expected fuel cost under the constraints of expected water available for hydrogeneration in a given period of time. The system variables are in discrete form, and the water inflows and load demand are stochastic. Duality decomposition [123] is applied to separate all the production units of the hydrothermal system thus giving two simpler subproblems for sequential optimisation. Thermal subproblem is solved using classical λ-dispatch procedure whereas hydrosubproblem is solved by first order conjugate gradient method.

No dimensionality problem arises in this algorithm and it can be used for multihydro and multithermal plants scheduling.

Soares et al [130] tackle the load demand uncertainties using the technique of stochastic programming. Instead of treating the load demand constraint of the problem a rigid constraint, the authors of the paper define a new decision variable which is the system total power supply. The objective is not only to find the optimal power generation levels at each production unit but also the optimal value of the total power supply. This is achieved by modifying the traditional objective function of the problem by augmenting it with the expected value of deficit and surplus of power supply over the random load demand. The modified objective function is optimised by forming the Lagrangian function which is defined as a function of hydrogeneration, thermal production, Lagrange multiplier and the new decision variable defining the system total power supply. Principle of duality decomposition is used to facilitate the partitioning of hydro and thermal generation which are then optimised as independent subsystems linked by a coupling equation. The proposed algorithm is illustrated through an example but is limited to an extent that transmission losses are not taken into account.

Kothari [131] presented an alternate approach to the stochastic optimal scheduling problem by considering the water storages at the end of each subinterval as independent variables inplace of water discharges. This formulation reduced the number of constraints resulting in reduced computer storage and computation time.

3.5 PROBABILISTIC FORMULATION

The review of literature reveals that most of the algorithms incorporate uncertainties in the system load demand and hydroreservoir water inflows but choose a deterministic objective functional representing thermal costs. A major source of uncertainty in optimal dispatch is that associated with cost coefficients a_i, b_i, and c_i [133]. Therefore, it is of utmost interest to refine hydrothermal optimal power dispatch procedures which can consider the following:

- Stochastic cost curves for thermal power generating units.
- Uncertainty in hydroreservoir water inflows.
- Uncertainty in system load demand.

The purpose of this section is to provide a technique which permits scheduling of long range hydrothermal system probabilistically considering above uncertainties. The approach is developed by posing a risk taking situation and considering thermal power generations P^m_{si}, water inflows J^m_j and load demand P^m_D during each subinterval as random variables. Any possible deviation of operating cost coefficients and load demand from their expected values is manipulated through the randomness of variable P^m_{si}.

With these variables stochastic, the optimisation is formulated as the minimisation of the sum of:

- Expected value of thermal fuel costs over the whole of the planning period

- Penalty costs associated with the expected value of the square of any possible deviations of the random variables from their respective expected values.

The objective function, therefore, can be written as

$$E[\sum_m \{\sum_{i=1}^{n} (a_i P_{si}^{m^2} + b_i P_{si}^m + c_i) + p(\bar{P}_D^m + \bar{P}_L^m - \sum_i P_{si}^m - \sum_j P_{H_j}^m)^2\}] \qquad (3.13)$$

The term $E[(\bar{P}_D^m + \bar{P}_L^m - \sum_i P_{si}^m - \sum_j P_{H_j}^m)^2]$ as derived in Appendix-C.1 simplifies to

$$\sum_{i=1}^{n} \text{var } P_{si}^m + \sum_{j=n+1}^{p} \sum_{v=n+1}^{p} \text{Cov}(P_{H_j}^m, P_{H_v}^m)$$

Variance of transmission loss has been neglected for convenience of analysis.

The optimisation problem is formulated as follows:

$$\text{Min } \bar{F}_T = \sum_m [\sum_{i=1}^{n} (\bar{a}_i \bar{P}_{si}^{m^2} + \bar{b}_i \bar{P}_{si}^m + \bar{c}_i + \bar{a}_i \text{ var } P_{si}^m)$$

$$+ p.\{\sum_{i=1}^{n} \text{var } P_{si}^m + \sum_{j\epsilon H} \sum_{v\epsilon H} \text{Cov}(P_{H_j}^m, P_{H_v}^m)\}] \qquad (3.14)$$

subject to:

i) Expected load demand constraint

$$\sum_{i\epsilon S} \bar{P}_{si}^m + \sum_{j\epsilon H} \bar{P}_{H_j}^m - \bar{P}_D^m - \bar{P}_L^m = 0 \qquad (3.15)$$

This constraint implies that the sum of the expected values of thermal power plant outputs and hydropower plant outputs must match the expected value of the sum of system transmission loss and system load demand at each subinterval. The expected transmission loss in sub-interval m is given in Appendix-C.

ii) Expected storage continuity constraint,

$$\bar{X}_j^{m+1} - \bar{X}_j^m - \bar{J}_j^m + Q_j^m = 0 \qquad (3.16)$$

iii) Total expected volume of water available constraint,

$$\bar{X}_j^{M+1} - X_j^1 - \sum_m \bar{J}_j^m + \sum_m Q_j^m = 0 \qquad (3.17)$$

iv) Expected hydropower generation constraint,

$$\bar{P}_{H_j}^m = h_j\{1+0.5c_j\ (2\bar{X}_j^m + \bar{J}_j^m - Q_j^m)\}\ (Q_j^m - \rho_j) \qquad (3.18)$$

$$Q_{jmin} \leq Q_j^m \leq Q_{jmax} \qquad (3.19)$$

$$\bar{P}_{si,min} \leq \bar{P}_{si}^m \leq P_{si,max} \qquad (3.20a)$$

$$\bar{P}_{H_{j,min}} \leq \bar{P}_{H_j}^m \leq \bar{P}_{H_{j,max}} \qquad (3.20b)$$

$$\bar{X}_{jmin} \leq \bar{X}_j^m \leq \bar{X}_j\ max \qquad (3.21)$$

X_j^1 is assumed to be known with probability one for $j \in H$ and expected final storage $\bar{X}_j^{M+1}$ is specified.

The probability properties of different quantities for hydroplants can be obtained as shown in Appendix-C.

3.5.1 Optimal Stochastic Control Strategy:

The problem has been solved by forming the Lagrangian function, obtained by augmenting the objective function of eqn.

(3.14) with various equality constraints expressed in terms of expected values, through the respective dual variables, as follows:

$$\alpha(\ldots) = \sum_m \Big[\sum_{i\varepsilon s} \{\bar{a}_i \bar{P}_{si}^{m^2} + \bar{b}_i \bar{P}_{si}^m + \bar{c}_i + (\bar{a}_i + p) c^2_{vP^m_{si}} \bar{P}_{si}^{m^2} \}$$

$$+ p \sum_{j\varepsilon H} \sum_{v\varepsilon H} \mathrm{Cov}(P_{Hj}, P_{Hv}) - \lambda_1^m \Big(\sum_i \bar{P}_{si}^m + \sum_j \bar{P}_{H_j}^m$$

$$- \bar{P}_L^m - \bar{P}_D^m \Big) + \sum_{j\varepsilon H} \lambda_{2j}^m \{ \bar{P}_{H_j}^m - h_j (1 + 0.5 c_j (2\bar{X}_j^m + \bar{J}_j^m - Q_j^m))\} (Q_j^m - \rho_j)$$

$$+ \lambda_{3j}^m (\bar{X}_j^{m+1} - \bar{X}_j^m - \bar{J}_j^m + Q_j^m) \Big] + \sum_{j\varepsilon H} \lambda_{4j} (\bar{X}_j^{M+1} - X_j^1 -$$

$$\sum_m \bar{J}_j^m + \sum_m Q_j^m) \qquad (3.22)$$

During each subinterval, the control variables are the water discharges through the turbines of the hydroelectric plants. The reservoir storages and the hydroelectric power generations at the end of a subinterval are obtained from eqns. (3.16) and (3.18) respectively. Irrespective of the actual hydrogenerations, the thermal generations satisfy eqn. (3.23) for minimal fuel cost and power transfer eqn. (3.15). These are taken as dependent variables.

The dual variables λ_1^m, λ_{2j}^m and λ_{3j}^m are obtained by equating the partial derivatives of the Lagrangian function with respect to the dependent variables to zero.

$$\left(\frac{\partial \alpha(\ldots)}{\partial \bar{P}_{si}^m}\right)_{i\varepsilon s} = \bar{b}_i + 2\bar{a}_i \bar{P}_{si}^m + (\bar{a}_i + p) . c^2_{vP_{si}} \, 2 \bar{P}_{si}^m -$$

$$\lambda_1^m \left(1 - \frac{\partial \bar{P}_L^m}{\partial \bar{P}_{si}^m}\right) = 0 \qquad (3.23)$$

$$\left(\frac{\partial \alpha(\ldots)}{\partial \bar{P}^m_{H_j}}\right)_{j\varepsilon H} = \lambda^m_{2j} - \lambda^m_1 \left(1- \frac{\partial \bar{P}^m_L}{\partial \bar{P}^m_{H_j}}\right) = 0 \qquad (3.24)$$

$$\left(\frac{\partial \alpha(\ldots)}{\partial \bar{X}^m_j}\right)_{\substack{j\varepsilon H \\ m\neq 1}} = \lambda^{m-1}_{3j} - \lambda^m_{3j} - \lambda^m_{2j}\; h_j\, c_j\, (Q^m_j - \rho_j) = 0 \qquad (3.25)$$

$\lambda^1_{3j} = 0$, since the equations associated with this set of dual variables are redundant.

With expected value of transmission loss as listed in Appendix-C-2, the gradient vector is,

$$\left(\frac{\partial \alpha(\ldots)}{\partial Q^m_j}\right)_{j\varepsilon H} = \lambda^m_{3j} + \lambda_{4j} - \lambda^m_{2j}\, h_j \{1+0.5c_j(2\bar{X}^m_j + \bar{J}^m_j - 2Q^m_j + \rho_j)\} + p\,\frac{d\,\mathrm{var}(P^m_{H_j})}{d\,Q^m_j} + 2\,p \sum_{\substack{v\varepsilon H \\ v\neq j}} \frac{d\,\mathrm{cov}(P^m_{Hj}, P^m_{Hv})}{d\,Q^m_j} +$$

$$\lambda^m_1\, B_{jj}\,\frac{d\,\mathrm{var}(P^m_{H_j})}{dQ^m_j} + 2\,\lambda^m_1 \sum_{\substack{v\varepsilon H \\ v\neq j}} B_{jv}\,\frac{d\,\mathrm{cov}(P^m_{H_j}, P^m_{H_v})}{dQ^m_j}$$

(3.26)

or

$$\left(\frac{\partial \alpha(\ldots)}{\partial Q^m_j}\right)_{j\varepsilon H} = \lambda^m_{3j} + \lambda_{4j} - \lambda^m_{2j}\, h_j \{1+0.5c_j(2\bar{X}^m_j + \bar{J}^m_j - 2\,\bar{Q}^m_j + \rho_j)\} + (p + \lambda^m_1\, B_{jj})\,\frac{d\,\mathrm{var}\,(P^m_{Hj})}{dQ^m_j}$$

$$+ \sum_{\substack{v\varepsilon H \\ v\neq j}} \left[2\{p + \lambda^m_1\, B_{jv}\}\,\frac{d\,\mathrm{cov}(P^m_{Hj}, P^m_{Hv})}{d\,Q^m_j}\right] \qquad (3.27)$$

where

$$\frac{d\ \mathrm{var}(P^m_{H_j})_{j\epsilon H}}{dQ^m_j} = 2h^2_j\ c^2_j\ (Q^m_j - \rho_j)\{\mathrm{var}\ (X^m_j) + 0.25\ \mathrm{var}(J^m_j)\}$$

(3.28)

and

$$\frac{d\ \mathrm{cov}(P_{H_j}, P_{H_v})}{dQ^m_j} = \sum_{v\epsilon H} h_j\ h_v\ c_j\ c_v(Q^m_v - \rho_v)\{\mathrm{cov}(X^m_j,\ X^m_v)$$

$$+0.25\ \mathrm{cov}\ (J^m_j,\ J^m_v\ )\} \qquad (3.29)$$

The dual variables λ_{4j} are to be adjusted to maximise the Lagrangian function of eqn. (3.22) under the constraints of other optimality conditions. The corresponding gradient vector is

$$\left(\frac{\partial\alpha(\ldots)}{\partial\lambda_{4j}}\right)_{j\epsilon H} = \bar{X}^{M+1}_j - X^1_j - \sum_m \bar{J}^m_j + \sum_m Q^m_j \qquad (3.30)$$

The upper and lower limits on the control variables are taken care of by making these variables equal to the respective bounded values whenever such limits are violated. For the dependent variables, these limits can be considered by augmenting the cost function through Powell's penalty function [132].

3.5.2 Computational Algorithm

In the previous sections the problem has been formulated and the optimal stochastic control policy outlined. Based upon this, the computational algorithm is illustrated in flow chart shown in Fig. 3.2. It starts with the assumption of a set of λ_{4j} and a feasible set of Q_j^m for hydroplants. The algorithm is based on conjugate gradient method employing decomposition technique.

3.6 SAMPLE SYSTEM STUDY

A long-range problem with two thermal generations and two hydroelectric plants is considered here.

Incremental costs of thermal plants:

$$\frac{\partial F_1}{\partial P_{s1}} = 0.80 + 0.04\ P_{s1}$$

$$\frac{\partial F_2}{\partial P_{s2}} = 0.78 + 0.06\ P_{s2}$$

$$C_{vP_{s1}} = 0.10$$

$$C_{vP_{s2}} = 0.10$$

penalty cost $p = 0.12$

Data in per unit for the two hydroplants are presented in Table 3.1. The water inflows and the standard deviations for the 6 subintervals are given in Table 3.2 The correlation coefficient between the water inputs for the same subintervals is assumed to be 0.5.

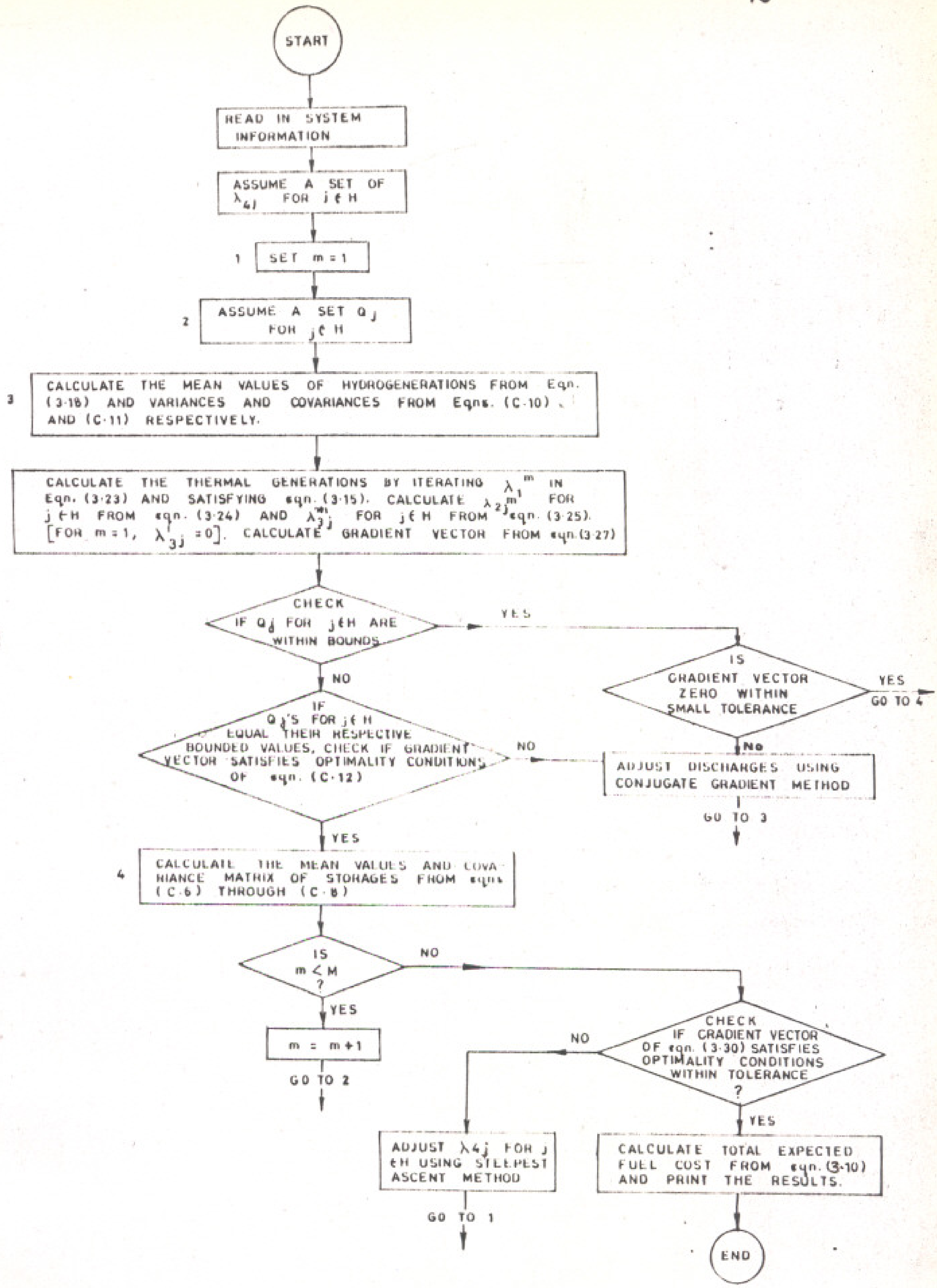

Fig. 3·2 Flow chart for the stochastic optimal scheduling of hydrothermal system.

Table 3.1: HYDROPLANT DATA IN PER UNIT

	Hydroplant 1	Hydroplant 2
h, c, ρ	1.0, 0.1, and 0.1 respectively	0.6, 0.12 and 0.05 respectively
Initial storages	10	12
Final Storages	10	12
Minimum and maximum allowable discharges	0.5, 3.5 respectively	0.5, 3.5 respectively

Table 3.2: WATER INFLOWS AND STANDARD DEVIATIONS IN PER UNIT

	m	1	2	3	4	5	6
Hydro-Plant-1	mean value	0	0.6	1.2	1.2	1.2	1.8
	standard deviation	0	0.12	0.24	0.24	0.24	0.36
Hydro-Plant 2	mean value	0	0	0	1.5	3.0	4.5
	standard deviation	0	0	0	0.3	0.6	0.9

The load demand is 8 p.u. and the loss formula matrix is

$$\begin{bmatrix} 0.05 & -0.02 & 0.01 & 0.00 \\ -0.02 & 0.06 & -0.02 & 0.01 \\ 0.01 & -0.02 & 0.04 & 0.00 \\ 0.00 & 0.01 & 0.00 & 0.02 \end{bmatrix}$$

With the initial values of variables assumed as given in Table 3.3, the optimal values obtained through the technique of Section 3.5.2 are given in Table 3.4. Table 3.5 gives the deterministic optimal values.

Table 3.3: INITIAL VALUES IN PER UNIT

Sub-Interval	Water discharge		Hydroelectric Power		Thermal Power	
	Q^m_1	Q^m_2	P^m_{H1}	P^m_{H2}	P^m_{S1}	P^m_{S2}
1.	0.9359	1.1883	1.6315	1.6178	2.7663	2.6672
2.	0.9672	1.1990	1.6374	1.5343	2.8073	2.7110
3.	1.0000	1.2500	1.6918	1.4965	2.7857	2.7116
4.	1.1540	1.2128	1.9942	1.4098	2.6030	2.6499
5.	1.2473	1.3592	2.1707	1.6782	2.3302	2.4444
6.	1.4515	2.2061	2.5773	3.0692	1.3574	1.6158

Table 3.4: STOCHASTIC OPTIMAL VALUES IN PER UNIT

Sub-Interval	Water discharge		Hydroelectric Power		Thermal Power	
	Q^m_1	Q^m_2	P^m_{H1}	P^m_{H2}	P^m_{S1}	P^m_{S2}
1.	0.7190	1.2031	1.2097	1.6382	3.0903	2.8192
2.	0.8073	1.2136	1.3566	1.5519	3.0194	2.8090
3.	0.9449	1.2525	1.6226	1.4970	2.8392	2.7372
4.	1.1024	1.2302	1.9427	1.4274	2.6320	2.6585
5.	1.0369	1.9523	1.8281	2.3911	2.2112	2.1856
6.	1.4266	2.2250	2.6239	2.9941	1.3622	1.6388

Fuel cost = 25.7663 p.u.

Table 3.5: DETERMINISTIC OPTIMAL VALUES IN PER UNIT

Sub-Interval	Water discharge		Hydroelectric Power		Thermal Power	
	Q_1^m	Q_2^m	P_{H1}^m	P_{H2}^m	P_{S1}^m	P_{S2}^m
1.	0.7454	1.1578	1.2668	1.5756	3.0724	2.8256
2.	0.8048	1.2139	1.3498	1.5560	3.0161	2.8040
3.	0.9554	1.2083	1.6400	1.4476	2.8459	2.7540
4.	1.0660	1.2250	1.8702	1.4289	2.6790	2.6807
5.	1.0806	1.9573	1.9110	2.4100	2.1359	2.1467
6.	1.3955	2.2452	2.5587	3.0345	1.3806	1.6399

Fuel cost = 25.6306 p.u.

3.7 CONCLUSIONS

The uncertainties present in water inflows, system load demand and operation cost equation coefficients affect the long range hydrothermal schedules. These uncertainties have been effectively accounted for by imposing risk taking situation. Any possible deviation of operating cost equation coefficients and system load demand from their respective expected values is manipulated through the randomness of generator powers. A procedure to obtain deterministic equivalent of stochastic formulation is outlined. The deterministic equivalent problem is solved using duality decomposition principle in order to obtain simpler subproblems. This makes the solution of the large scale hydrothermal system possible. The decomposed thermal sub-problem is solved by classical λ-dispatch and hydrosubproblem is solved using conjugate gradient method of Fletcher and Reeves. The method has reasonable rate of convergence and modest storage requirements.

The practical viability of the formulation is illustrated through a numerical example. Because of the tremendous annual fuel cost in thermal plants, a small percentage saving can be considered to be significant, thus justifying the need for more accurate analysis and consideration of randomness in variables. It is noticed in the numerical example that the optimal power generations of each plant are considerably different in various sub-intervals, even though the expected load demands are the same in each sub-interval.

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CHAPTER - 4

STOCHASTIC OPTIMAL HYDROTHERMAL SCHEDULING WITH CASCADED PLANTS

4.1 INTRODUCTION

In Chapter 3, a novel approach for incorporating uncertainties in water inflows, system load demand and operating cost coefficients has been developed for the solution of hydrothermal scheduling problem where the hydro reservoirs are considered to be independent of each other. Scheduling of cascaded plants in another important problem in hydrothermal systems. The motivation in this chapter is to demonstrate the generality, effectiveness and practical applicability of the novel approach of chapter 3 by applying it to hydrothermal scheduling with cascaded plants.

4.2 SCHEDULING WITH CASCADED PLANTS

In practical power systems, situations exist where there are several cascaded reservoirs on the same stream. The problem becomes much more complicated because of a time delay involved in water transport from upstream to the downstream reservoirs. In such a situation discharge in a particular sub-interval from an upstream hydroplant not only affects the schedule in that particular subinterval but also the schedule in some later subintervals depending upon the water transport time delay. If there is a series of cascaded plants on the same stream, then a change in discharge for the first upstream reservoir will affect the schedule in many subsequent subintervals thereby increasing complexity of the problem.

The problem of hydrothermal scheduling with cascaded plants has been investigated by many researchers[65, 79, 85, 134-136] using different techniques. All these techniques used deterministic input data. To the best of the author's knowledge Nagrath & Kothari [137] were the first to tackle the problem of cascaded plants with probabilistic data. They considered uncertainties in water inflows and load demand and used nonlinear programming approach alongwith gradient technique for finding the stochastic optimal schedules. Soares et al [130] applied a decomposition and coordination technique to deal with optimal operation of large scale hydrothermal system consisting of general hydraulic networks with cascade plants, time delays and spilling. They consider stochastic load demand but assume deterministic water inflows. Their approach has been briefly reviewed in Chapter 3.

In the next section a probabilistic formulation of the problem is discussed which takes into account uncertainties in (i) reservoir water inflows, (ii) operation cost coefficients, and (iii) system load demand.

4.3 PROBABILISTIC FORMULATION

A few justifiable assumptions are required to be made to make the problem more tractable. These are:

(i) Water inflows into various reservoirs on different streams are statistically correlated during the same subinterval but are independent during different subintervals.

(ii) The power demand on the system is assumed to be statistically independent of water inflows.

(iii) The mean values and covariance matrix of the water inflows are known from the past history.

These assumption are fairly common in the Water Management Problems.

In the proposed probabilistic formulation, the input variables representing the water inflows, cost coefficients and system load demand are taken as random variables. A risk taking situation is posed in the problem. Any possible deviations of random variables representing cost coefficients and system load demand from their respective expected values are manipulated by treating variables P_{si}, representing thermal power generations as random variables.

The objective in stochastic hydrothermal scheduling problem is thus, to minimise the value of the objective function $\bar{F}_T$ representing the combined system cost consisting of the sum (i) expected value of operating cost of thermal generations, and (ii) expected cost of deviations, a penalty term proportional to the expectation of the square of unsatisfied load because of the possible variance associated with random variables P_{si}

and P_{Hj}. The minimisation is carried over the planning period of one year, while utilising the expected volume of water available over the planning period, and satisfying the expected power demand on the power system during each subinterval of time.

Mathematically, the problem can be stated as

Minimise

$$E[\sum_m \{\sum_{i=1}^{n} (a_i P_{si}^{m^2} + b_i P_{si}^m + c_i) + p(\bar{P}_D^m + \bar{P}_L^m - \sum_i P_{si}^m - \sum_j P_{Hj}^m)^2\}] \quad (4.1a)$$

As shown in Chapter 3 (eqn. 3.14), the expected cost during a subinterval m is given as

$$\bar{F}_T = \sum_m [\sum_{i=1}^{n} (\bar{a}_i \bar{P}_{si}^m + \bar{b}_i \bar{P}_{si}^m + \bar{a}_i \text{ var } P_{si}^m) + p\{\sum_{i \in s} \text{var } P_{si}^m + \sum_{(jUjo)\in H} \sum_{(vUv_o)\in H} \text{cov}(P_{Hj}^m, P_{Hv}^m)\}] \quad (4.1b)$$

subject to

(i) $\sum_i \bar{P}_{si}^m + \sum_{(jUjo)\in H} \bar{P}_{Hj}^m - \bar{P}_D^m - \bar{P}_L^m = 0$ (4.2)

This constraint implies that the sum of the expected values of thermal power plant outputs and hydropower plant outputs must match the expected value of the sum of system transmission loss and system load demand during each subinterval. The expected transmission loss in subinterval m is given in Appendix-C.

$$\bar{P}_L = \sum_{i\epsilon s} \sum_{j\epsilon s} \bar{P}^m_{si} B_{ij} \bar{P}^m_{sj} + \sum_{(uUu_o)\epsilon H} \sum_{(vUv_o)\epsilon H} \bar{P}_{Hu} B_{uv} \bar{P}^m_{Hv}$$

$$+2 \sum_{j\epsilon s} \sum_{(vuv_o)\epsilon H} \bar{P}^m_{sj} B_{jv} \bar{P}^m_{Hv} + \sum_{i\epsilon s} B_{ii} \text{ var } P^m_{si}$$

$$+ \sum_{(jUj_o)\epsilon H} \sum_{(vUv_o)\epsilon H} B_{jv} \text{ cov } (P^m_{Hj}, P^m_{Hv}) \qquad (4.3)$$

(ii) Expected hydropower generation constraints

$$\bar{P}^m_{Hj} = h_j \{1+0.5c_j (2\bar{X}^m_j + \bar{J}^m_j - Q^m_j)\} (Q^m_j - \rho_j) \qquad (4.4)$$

and

$$\bar{P}^m_{Hjo} = h_{jo} \{1+0.5c_{jo} (2\bar{X}^m_{jo} + \bar{J}^m_{jo} + Q^{m-\tau}_j - Q_{jo})\}(Q^m_{jo} - \rho_{jo}) \qquad (4.5)$$

where jo is the index of the first neighbouring downstream reservoir to plant j.

The expected storage constraints

$$\bar{X}^{m+1}_j - \bar{X}^m_j - \bar{J}^m_j + Q^m_j = 0 \qquad (4.6)$$

$$\bar{X}^{m+1}_{jo} - \bar{X}^m_{jo} - \bar{J}^m_{jo} - Q^{m-\tau}_j + Q^m_{jo} = 0 \qquad (4.7)$$

Total expected volume of water availability constraints.

$$\bar{X}^{M+1}_j - \bar{X}^1_j - \sum_m \bar{J}^m_j + \sum_m Q^m{}_j = 0 \qquad (4.8)$$

$$\bar{X}^{M+1}_{jo} - \bar{X}^1_{jo} - \sum_m \bar{J}^m_{jo} - \sum_m Q^m_j + \sum_m Q^m_{jo} = 0 \qquad (4.9)$$

$$Q_{jmin} \leq \bar{Q}_j^m \leq Q_{jmax} \tag{4.10}$$

$$\bar{P}_{si,min} \leq \bar{P}_{si}^m \leq \bar{P}_{si,max} \tag{4.11}$$

$$\bar{X}_{jmin} \leq \bar{X}_j^m \leq \bar{X}_{j\,max} \tag{4.12}$$

X_j^1 is assumed to be known with probability 1 for $j\varepsilon H$ and expected final storage $\bar{X}_j^{M+1}$ is specified. The probability properties of different quantites of hydroplants can be obtained as shown in Appendix-D.

4.4 METHOD OF SOLUTION

The problem has been solved by forming the Lagrangian function, obtained by augmenting the objective function of eqn. (4.1b) with various equality constraints expressed in terms of expected values, through the respective dual variables, as follows:

$$\alpha(\ldots) = \sum_m \Big[\sum_{i\varepsilon s} \{\bar{a}_i \bar{P}_{si}^{m^2} + \bar{b}_i \bar{P}_{si}^m + (\bar{a}_i + p)\, C_{vP_{si}}^2 \bar{P}_{si}^{m^2}\}$$

$$+ p \sum_{(jUjo)\varepsilon H} \sum_{(vUv_o)\varepsilon H} \mathrm{cov}(P_{Hj}^m, P_{Hv}^m) - \lambda_1^m \Big(\sum_{i\varepsilon s} \bar{P}_{si}^m$$

$$+ \sum_{(jUj_o)\varepsilon H} \bar{P}_{Hj}^m - \bar{P}_L^m - \bar{P}_D^m \Big) + \sum_{j\varepsilon H} \lambda_{2j}^m \big[\bar{P}_{Hj} - h_j \{1 +$$

$$0.5 c_j (2\bar{X}_j^m + \bar{J}_j^m - \bar{Q}_j^m)\} (\bar{Q}_j^m - \rho_j) \big] + \sum_{jo\varepsilon H} \lambda_{2jo}^m \big[\bar{P}_{Hjo}^m -$$

$$h_{jo}\{1+0.5c_{jo}(2\bar{X}^m_{jo} + \bar{J}^m_{jo} + Q^{m-\zeta}_j - Q^m_{jo})\}\ (Q^m_{jo} - \rho_{jo})]$$

$$+ \sum_{j\epsilon H} \lambda^m_{3j}\ (\bar{X}^{m+1}_j - \bar{X}^m_j - \bar{J}^m_j + Q^m_j) + \sum_{jo\epsilon H} \lambda^m_{3jo}\ (\bar{X}^{m+1}_{jo}$$

$$- \bar{X}^m_{jo} - \bar{J}^m_{jo} - Q^{m-\zeta}_j + Q^m_{jo})] + \sum_{j\epsilon H} \lambda_{4j}\ (\bar{X}^{M+1}_j - \bar{X}^1_j - \sum_m \bar{J}^m_j + \sum_m Q^m_j)$$

$$+ \sum_{jo\epsilon H} \lambda_{4jo}\ (X^{M+1}_{jo} - X^1_{jo} - \sum_m \bar{J}^m_{jo} - \sum_m Q^{m-\zeta}_j + \sum_m Q^m_{jo})$$

(4.13)

The control variables, during each subinterval, are the water discharges, and rest of the variables are the dependent variables. The dual variables λ^m_1, λ^m_{2j}, λ^m_{2jo}, λ^m_{3j} and λ^m_{3jo} are obtained by equating the partial derivatives of the Lagrangian function with respect to the dependent variables to zero.

$$\left(\frac{\partial\alpha(\ldots)}{\partial P^m_{si}}\right)_{i\epsilon s} = 2\bar{a}_i\ \bar{P}^m_{si} + \bar{b}_i + 2(\bar{a}_i + p)\ C^2_{vP_{si}}\ \bar{P}^m_{si} -$$

$$\lambda^m_1\left(1 - \frac{\partial \bar{P}^m_L}{\partial \bar{P}^m_{si}}\right) = 0 \qquad (4.14)$$

$$\left(\frac{\partial\alpha(\ldots)}{\partial \bar{P}^m_{Hj}}\right)_{(jUjo)\epsilon H} = \lambda^m_{2j} - \lambda^m_1\left(1 - \frac{\partial \bar{P}^m_L}{\partial P^m_{Hj}}\right) = 0 \qquad (4.15)$$

$$\left(\frac{\partial\alpha(\ldots)}{\partial \bar{X}^m_j}\right)_{\substack{(jUjo)\epsilon H \\ m\neq 1}} = \lambda^{m-1}_{3j} - \lambda^m_{3j} - \lambda^m_{2j}\ h_j\ c_j(Q^m_j - \rho_j) = 0$$

(4.16)

$\lambda^1_{3j} = 0$, since the equations associated with the set of dual variables are redundant.

The gradient vector is

$$\left(\frac{\partial\alpha(\ldots)}{\partial Q_j^m}\right)_{j\epsilon H} = \lambda_{3j}^m + \lambda_{4j} - \lambda_{2j}^m h_j\{1+0.5c_j(2\bar{X}_j^{m+1} + \bar{J}_j^m -$$

$$2Q_j^m + \rho_j)\} + p\frac{d\ var\ (P_{Hj}^m)}{dQ_j^m} + 2p\sum_{\substack{(vUv_o)\epsilon H \\ v\neq j}} \frac{d\ cov(P_{Hj}^m, P_{Hv}^m)}{dQ_j^m}$$

$$+\lambda_1^m\ B_{jj}\ \frac{d\ var\ (P_{Hj}^m)}{d\ Q_j^m} + 2\lambda_1^m \sum_{\substack{(vUv_o)\epsilon H \\ v\neq j}} B_{jv}\ \frac{d\ cov(P_{Hj}^m, P_{Hv}^m)}{dQ_j^m}$$

$$-\lambda_{3jo}^m - \lambda_{4jo} - \lambda_{2jo}^m h_{jo}\{1+0.5c_{jo}(Q_{jo}^{m-\tau} - \rho_{jo})\}$$

or

$$\left(\frac{\partial\alpha(\ldots)}{\partial Q_j^m}\right)_{j\epsilon H} = \lambda_{3j}^m + \lambda_{4j} - \lambda_{2j}^m h_j\{1+0.5c_j(2\bar{X}_j^{m+1} + \bar{J}_j^m$$

$$-2Q_j^m + \rho_j)\} + (p + \lambda_1^m\ B_{jj})\frac{d\ var\ (P_{Hj}^m)}{d\ Q_j^m} + \sum_{\substack{(vUv_o)\epsilon H \\ v\neq j}} 2\{p +$$

$$\lambda_1^m\ B_{jv}\}\ \frac{d\ cov\ (P_{Hj}^m, P_{Hv}^m)}{d\ Q_j^m} - \lambda_{3jo}^m - \lambda_{4jo} - \lambda_{2jo}^m h_{jo}$$

$$\{1+0.5c_{jo}(Q^{m-\tau} - \rho_{jo})\} \qquad (4.17)$$

$$\left(\frac{\partial\alpha(\ldots)}{\partial Q_{jo}}\right)_{j_o\epsilon H} = \lambda_{4jo} + \lambda_{3jo}^m - \lambda_{2jo}^m h_{jo}\{1+0.5c_{jo}(2\bar{X}_{jo}^m +$$

$$\bar{J}_{jo}^m + Q_j^{m-\tau} - 2Q_{jo}^m + \rho_{jo})\} + (p + \lambda_1^m\ B_{jo,jo})\frac{d\ var(P_{Hjo}^m)}{d\ Q_{jo}^m}$$

$$+\sum_{\substack{(vUv_o)\epsilon H \\ v\neq j_o}} 2\{p + \lambda_1^m\ B_{jo,v}\}\ \frac{d\ cov(P_{Hjo}^m, P_{Hv}^m)}{d\ Q_{jo}^m} \qquad (4.18)$$

The dual variables λ_{4j} and λ_{4jo} are to be adjusted to maximise the Lagrangian function of eqn. (4.13) under the constraints of other optimality conditions. The corresponding gradient vector is

$$\left(\frac{\partial\alpha(\ldots)}{\partial\lambda_{4j}}\right)_{j\epsilon H} = \bar{X}_j^{M+1} - X_j^1 - \sum_m \bar{J}_j^m + \sum_m Q_j^m \qquad (4.19)$$

$$\left(\frac{\partial\alpha(\ldots)}{\partial\lambda_{4jo}}\right)_{jo\epsilon H} = \bar{X}_{jo}^{M+1} - X_{jo}^1 - \sum_m \bar{J}_{jo}^m - \sum_m Q_j^m + \sum_m Q_{jo}^m \qquad (4.20)$$

4.5 STOCHASTIC SCHEDULING ALGORITHM

(a) Assume a set of λ_{4j} for $(j \cup jo)\epsilon H$

(b) Set m=1

(c) Assume a set of Q_j for $(j \cup jo)\epsilon H$.

(d) Calculate the expected hydrogenerations from eqns. (4.4) and (4.5) and the covariance matrix from eqns. (D.11) and (D.12).

(e) Assume λ_1^m . Calculate the thermal generations from eqn. (4.14) by Gauss-Seidel iterative method. Calculate expected transmission loss from eqn. (4.3).

(f) Check for power transfer eqn. (4.2). If this is not satisfied within prescribed accuracy, repeat from step (e) for a different value of λ_1^m till power-transfer eqn. (4.2) is nearly satisfied.

(g) Calculate λ_{2j}^{m} for $(j \cup jo) \varepsilon H$ from eqn. (4.15), λ_{3j}^{m} for $(j \cup jo) \varepsilon H$ and $m > 1$ from eqn. (4.16), and the gradient vector from eqns. (4.17) and (4.18). If this does not satisfy the optimality conditions given by eqn. (C.12) within a prescribed accuracy, adjust the water discharges using first order gradient method and repeat from step (d).

(h) Calculate the expected values and covariance matrix of storages from eqns. (D.5) and (D.6)

(i) If $m < M$, substitute $m = m+1$ and repeat from step (c).

(j) The solution, although it satisfies optimality conditions, may not satisfy eqns. (4.8) and (4.9). Adjust λ_{4j} for $(j \cup jo) \varepsilon H$ by the first order gradient method with gradients obtained from eqns. (4.19) and (4.20). Repeat the procedure from step (b) until eqns. (4.8) and (4.9) are satisfied within prescribed accuracy.

4.6 SAMPLE SYSTEM STUDY

The problem with one equivalent thermal plant and two series hydroplants (i.e. on the same water stream) is considered here. The system is shown in Fig. 4.1.

Incremental cost of thermal plant

$$\frac{dF}{dP_{s1}} = 0.8 + 0.1 \, P_{s1}$$

Data in p.u. for the two hydroplants are given in Table 4.1. The water inflows and the standard deviations for the six subintervals are given in Table 4.2.

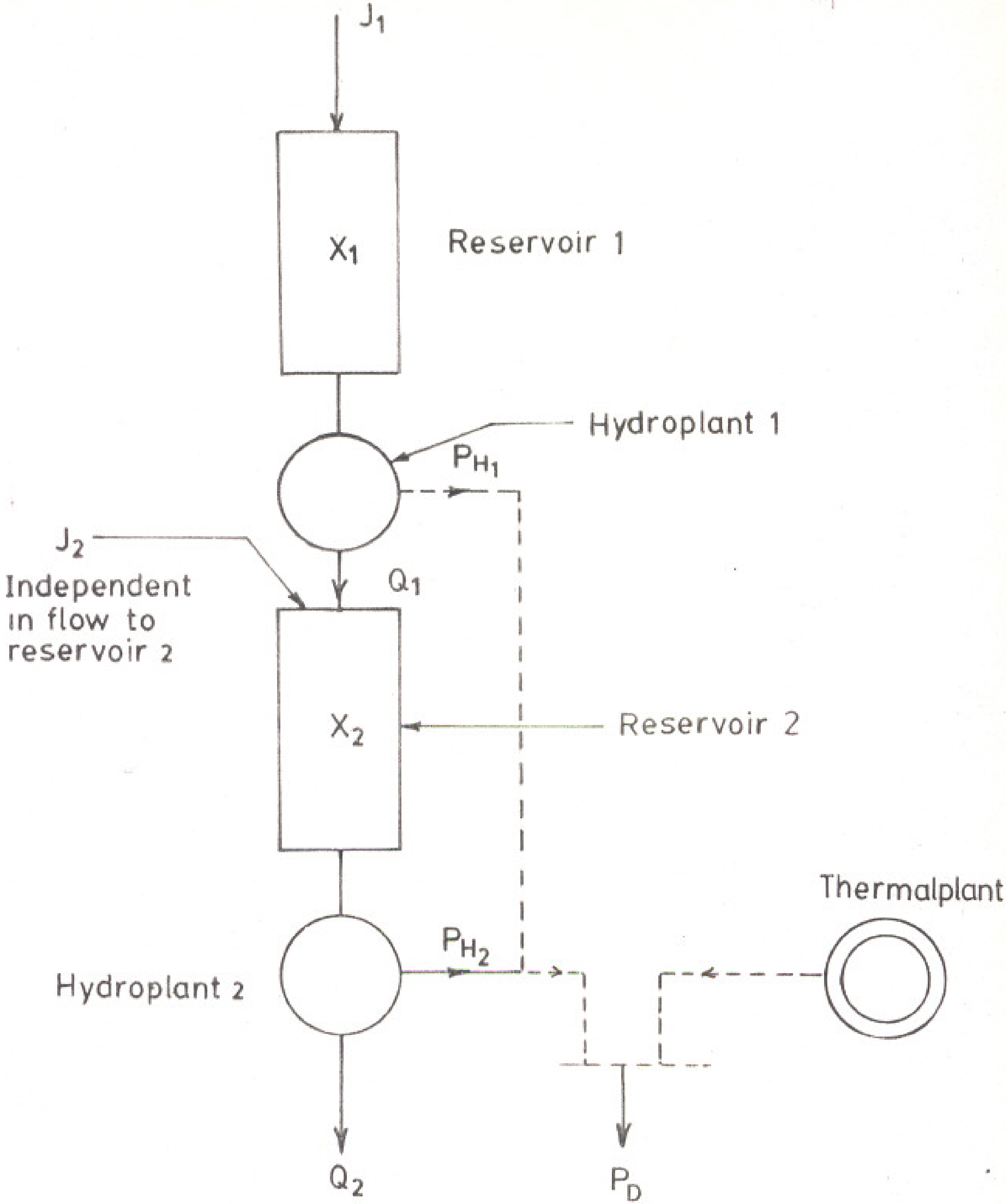

Fig.4.1. Schematic diagram of a cascaded hydrothermal system.

The correlation coefficient between the water inputs for the same subintervals is assumed to be 0.5. No time delay is assumed, i.e., $\tau = 0$. The load demand is 10.0 p.u. and its standard deviation is 1.0 p.u.

Table 4.1: HYDROPLANT DATA, IN PER UNIT

	Hydroplant 1	Hydroplant 2
h, c, ρ	0.5, 0.05, and 0.1 respectively	0.4, 0.06 and 0.1 respectively
Initial storages	25.0	25.0
Final storages	20.5	20.5
Minimum and Maximum allowable discharges	1.0 and 4.0 respectively	1.0 and 4.0 respectively

The loss formula matrix is:

$B_{ij} = 0.005$ for $i = j$

$B_{ij} = 0.0$ for $i \neq j$ $i, j = 1, 2, 3$

Table 4.2: WATER INFLOWS AND STANDARD DEVIATIONS, IN PER UNIT.

	m	1	2	3	4	5	6
Hydroplant 1	Mean value	1.0	2.0	3.0	3.0	2.0	1.0
	Standard deviation	0.04	0.16	0.36	0.36	0.16	0.04
Hydroplant 2	Mean value	0.5	0.5	0.5	0.5	0.5	0.5
	Standard deviation	0.01	0.01	0.01	0.01	0.01	0.01

The optimal solution is presented in Fig. 4.2.

4.7 CONCLUSIONS

The novel approach developed in Chapter 3 has been successfully extended to hydrothermal systems with cascaded plants. Numerical results are presented in Fig.4.2. The generality of the stochastic optimal power dispatch is established. Inaccuracies in the operating cost coefficients have been tackled for the first time for solving the cascaded problem.

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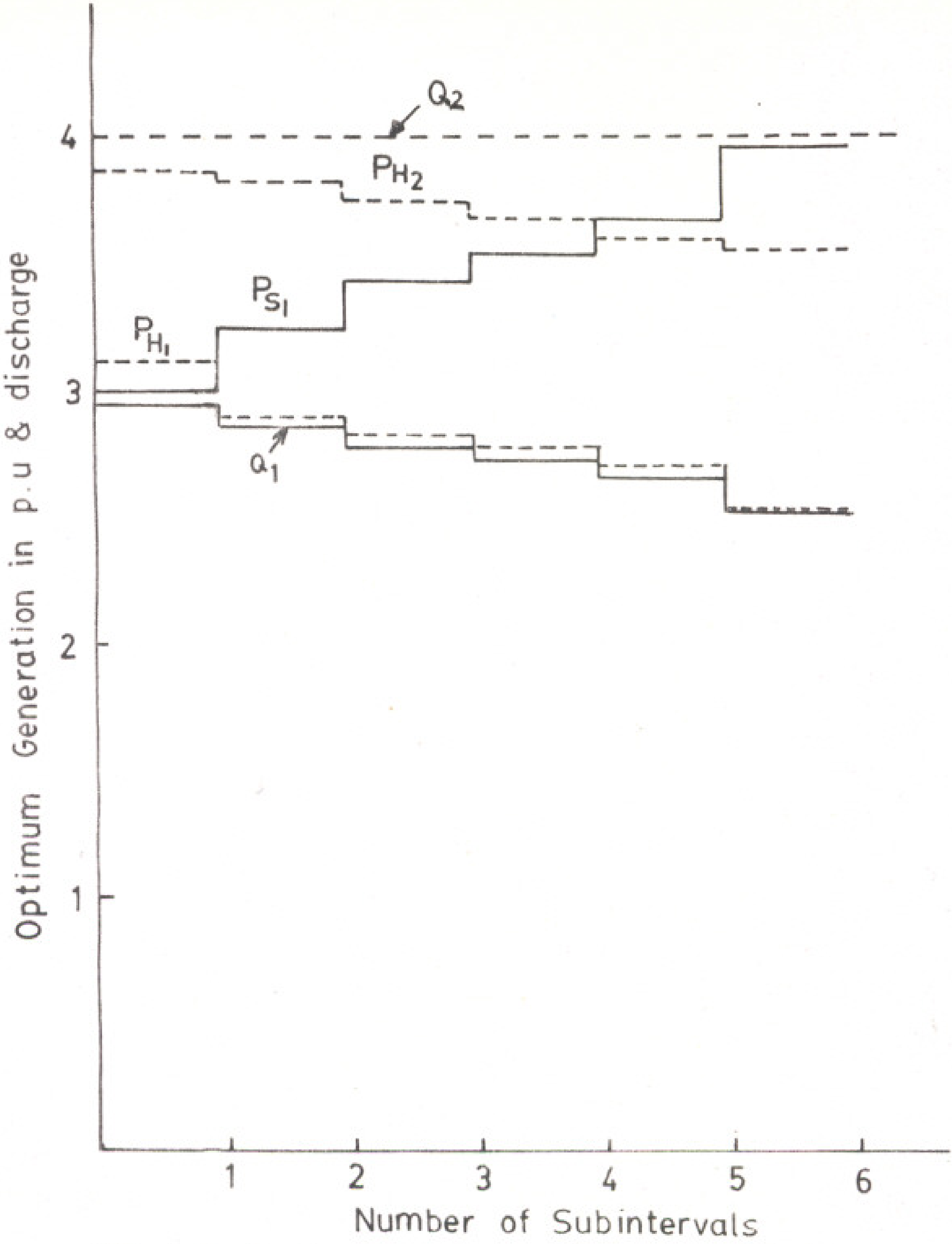

Fig 4.2. Optimal values in per unit for cascaded hydro system.

CHAPTER - 5

HYDROTHERMAL SCHEDULING USING FLETCHER'S TECHNIQUE

5.1 INTRODUCTION

Variable metric minimisation algorithms, also called quasi-Newton iteration algorithms, have proved to be the most effective class of general purpose methods for solving unconstrained minimisation problems. Their development is continuing so rapidly, however, that the vast array of possibilities open to a researcher wishing to implement such a method is daunting. Such minimisation algorithms require the determination of the step size accurately in each iteration such that maximum reduction in the function value is achieved during each iteration. Powell [138] establishes the superlinear convergence of quasi-Newton methods with exact line searches for a function where Hessian matrix is positive definite everywhere. Quite obviously accurate minimisation along each search direction can be expensive in function evaluations and their gradients. Much more work recently has been devoted to the question of convergence of the methods when the linear searches are dropped in favour of stability requirement: that function must decrease at every step.

Under certain conditions on the second derivatives including positive definitness of the Hessian at the solution Broyden, Dennis and More [139] established superlinear convergence when each step length is taken as unity. Powell [140] establishes convergence of BFGS algorithm without line searches for arbitrary convex functions and that this convergence is superlinear if the Hessian matrix at the solution is positive definite.

The common procedure now is to try the full quasi-Newton step first and if this unity step length fails to satisfy the criterion in use, to backtrack in a systematic way along the direction defined by the step. Computational experience has shown the importance of taking a full quasi-Newton step whenever possible. Failure to do so leads forfeiture of the advantage of Newton's method near the solution.

This chapter outlines the underlying structure of Fletcher's technique [141] which avoids an inverse interpolation in performing the linear search and suggests an acceptable point search procedure which takes the first point in some generated sequence which satisfies some acceptance criterion. The technique is then applied to the problem of optimal hydrothermal generation scheduling. The review of literature reveals that Wadhwa et al [142] were the first to apply Fletcher's technique to the solution of electrical power system operation problems. They successfully applied the algorithm to the optimal load flow solution. Recently, Bhatele et al [143] have used this algorithm in reactive power optimisation.

5.2 FLETCHER'S TECHNIQUE

The importance of the full linear search is that it furnishes a property which enables finite termination to be proved for quadratic functions. Obviously the special properties of the conjugate gradient and quasi-Newton algorithms are related to their use on quadratic functions, and for such functions a single step resulting from a quadratic or cubic fit attains the minimum along the line. Thus the quadratic terminal convergence will not be impaired if only a single such fit is made at each iteration, and since the special properties do not apply in a non-quadratic region nothing is lost by not continuing the minimum in this case. Fletcher's technique gives weak step length $\{\alpha^k\}$ acceptance criterion that performs well both in theory and practice. Fletcher's technique solves the problem of minimising a non-linear function subject to the availability of gradient g and non-availability of Hessian G. Fletcher replaced the property of quadratic termination by a property which requires that for a quadratic function with exact Hessian $G(=\nabla^2 F(X^k))$ strictly positive definite, the eigenvalues of an approximating Hessian H to G must tend monotonically to those of G^{-1} in certain sense (he calls this "Property 1"). The success of Fletcher's technique is attributed to Fletcher and Powell updating formula[144] satisfying Property 1 of Fletcher's technique.

Fletcher's technique solves the problem of minimising a nonlinear function subject to the availability of a gradient g and nonavailability of a Hessian G.

Let x^k be any point at kth iteration of the solution. Expanding the function around x^k and using Taylor's series upto first order term

$$F(X^{k+1}) = F(X^k + \Delta X) = F(X^k) + \nabla^T F \ \Delta X + \ldots \qquad (5.1)$$

$$\Delta F = F(X^{k+1}) - F(X) = \nabla^T F \ \Delta X$$

$$\Delta F = g^T t \qquad (5.2)$$

where, $g^T(X^k) = \nabla^T F(X^k)$ and $t = \Delta X = X^{k+1} - X^k$

In Fletcher's technique linear search is dispensed with. The abandonment of linear search requires that something is done to force a sufficient large decrease in F at each iteration to guarantee ultimate convergence. However, to ensure the efficiency of the algorithm without linear searches, it is necessary that only one evaluation each of F and g be made in an iteration, except on rare occasions [141]. Many suggestions have been made for the method of choosing step length α^k. The requirement that $F(X^{k+1}) < F(X^k)$ is not sufficient on its own. Most use the fact that

$$\frac{F(X^{k+1}) - F(X^k)}{g^T t} \rightarrow 1 \quad \text{as } \alpha^k \rightarrow 0 \qquad (5.3)$$

to ensure that the step length is not too small and so prevent premature convergence. Similarly the decrease in the value of the function should be sufficient to ensure that the line minimum has not been too wildly overestimated. Equation (5.2) implies that it is not desirable that F be very different in magnitude from $g^T.t$. Fletcher's [141] original suggestion is that by choosing successively

$$\alpha^k = 10^{-j} \qquad j=0,1,\ldots, \tag{5.4}$$

the change in F relative to $g^T t$ cannot become arbitrarily small if

$$0 < \text{tolerance} \leq \frac{\Delta F}{g^T . t} \quad \text{for tolerance} << 1 \tag{5.5}$$

unless the minimum has been found.

This presumes

$$g^T t < 0 \tag{5.6}$$

to ensure 'downhill' direction. Fletcher recommends the value of tolerance equal to 0.0001. However, if the point is not acceptable, he uses a cubic inverse interpolation to reduce α, with the proviso that 0.1α be the smallest value permitted to generate the next step in the search.

In any practical program a test of the condition given by eqn. (5.6) is advisable before proceeding with any search along t. It is also desirable to insist that

$$(t^k)^T \; y^k > 0 \tag{5.7}$$

where $y^k = g^{k+1} - g^k$

since this guarantees the positive definiteness of H^{k+1}.

In Fletcher's technique, the direction vector d is given by

$$d = - Hg \tag{5.8a}$$

and the correction t is determined by

$$t = -\alpha \, Hg \tag{5.8b}$$

where the value of α is chosen to satisfy inequality (5.5).

Fletcher is confident that choice of $\alpha=1$ is not too small. Therefore, he tries the estimates of α as given by eqn. (5.4) and he accepts the first estimate that satisfies inequality of expression (5.5) with choice of tolerance [141] equal to 0.0001. In practice, he finds that in most iterations the value of $\alpha=1$ is satisfactory. However, no guarantee can be given that these values could be the optimum when applied to power systems.

Goldstein and Price [145] have shown that if the approximate inverse Hessian H tends to the actual inverse Hessian G^{-1} when converging to the minimum of a non-quadratic function then eventually towards the end of the optimisation the choice of $\alpha=1$ will always be preferred and convergence will be superlinear. It is expected that this behaviour of the technique will occur in most cases and so the efficiency of the method in eventually requiring only one evaluation of F and g per iteration is justified.

The only other requirement imposed by the algorithm is that H should be positive definite. Consequently, the most common choice is to set H=I (Identity matrix) and this can usually be expected to perform well. 'Property 1' defined by Fletcher as has already been mentioned requires that for quadratic functions the eigenvalues of H must tend monotonically to those of G^{-1} and it is proved that FP updating formula given by eqn. (5.9) satisfies this property and, therefore, becomes a suitable candidate to update H in the algorithm envisaged [141].

$$H^{k+1} = H^{k} + \frac{(t^{k})^{T}\ t^{k}}{(t^{k})^{T}\ y^{k}} - \frac{(H^{k})^{T}\ y^{k}(\ y^{k})^{T}\ H^{k}}{(\ y^{k})^{T}\ H^{k}\ y^{k}}\ . \tag{5.9}$$

However, the updating of Hessian as given in FP method makes the matrix singular for large systems as the optimal solution is approached. Therefore, it is desirable to find out some other method of updating.

The updating formula due to FP maps y into t in the H space i.e. $H^{k+1}\ y^{k} = t^{k}$.

Fletcher's updating formula is based on the recursion relation for inverting the matrices and is given below:

$$(H^{k+1})^{-1} = [\,I - \frac{y^{k}(t^{k})^{T}}{(t^{k})^{T}\ y^{k}}\,]\,[\,H^{k}\,]^{-1}\,[\,I - \frac{t^{k}\ (y^{k})^{T}}{(t^{k})^{T}\ y^{k}}\,] + \frac{y^{k}(y^{k})^{T}}{(t^{k})^{T}\ y^{k}} \tag{5.10}$$

This relation forces $(H^{k+1})^{-1}\ t^{k} = y^{k}$.

Now a simple interchange between t and y i.e. $t \rightarrow y$ is made in eqn. (5.10) and a formula for updating H is obtained as

$$H^{k+1} = [\,I - \frac{t^{k}(y^{k})^{T}}{(t^{k})^{T}\ y^{k}}\,]\ H^{k}\ [\,I - \frac{y^{k}(t^{k})^{T}}{(t^{k})^{T}\ y^{k}}\,] + \frac{t^{k}(t^{k})^{T}}{(t^{k})^{T}\ y^{k}}$$

$$= H^{k} - \frac{t^{k}(y^{k})^{T}\ H^{k}}{(t^{k})^{T}\ y^{k}} - \frac{H^{k}\ y^{k}(t^{k})^{T}}{(t^{k})^{T}\ y^{k}} + (1 + \frac{(y^{k})^{T}H^{k}\ y^{k}}{(t^{k})^{T}\ y^{k}})\,(\frac{t^{k}(t^{k})^{T}}{(t^{k})^{T}\ y^{k}}) \tag{5.11}$$

It is seen here that the correction to the Hessian is done in a space that is spanned by t and Hy. The use of the FP formula for updating as is mentioned earlier results in singularity for large systems as the optimal solution is reached and if formula given by eqn. (5.11) is used alone then H may tend to unboundedness. Therefore, a combination of two formulae given by eqns. (5.9) and (5.11) is necessary. In Fletcher's technique updating of Hessian is done by making use of either eqn. (5.9) or eqn. (5.11) depending upon certain criterion. This criterion keeps the Hessian away from singularity and unboundedness.

If $(y^k)^T\ t^k < (y^k)^T\ H^k\ y^k$ use eqn. (5.9)

$(y^k)^T\ t^k \geq (y^k)^T\ H^K\ y^k$ use eqn. (5.11)

It is seen that in the initial stages updating by eqn. (5.9) behaves nicely and as the process tends towards optimal solution and Hessian exhibits singularity, the updating is switched over according to eqn. (5.11). The updating scheme is such that for quadratic functions the eigenvalues of H tend to those of G^{-1} which guarantees positive difiniteness of the matrix H in successive iterations.

The Fletcher's technique, therefore, has the following characteristics:

i) The linear search is avoided;

ii) The property of quadratic termination, whose relevance for general functions has always been questionable, has been replaced by another property in which the approximation to inverse Hessian tends to the inverse Hessian G^{-1} by updating the H matrix according to a particular strategy with a convex class of formulae.

5.3 PROBLEM FORMULATION

The optimal power dispatch problem is to minimise the cost of generation for a given load demand subject to system equality and inequality constraints. The required deterministic problem formulation of hydrothermal optimal power dispatch has already been discussed in Section 3.3. The constrained minimisation problem is transformed to an unconstrained one by using Lagrange multiplier technique. The problem has been solved by forming the Lagrangian function, obtained by augmenting the objective fuction of eqn. (3.10) with equality constraints of eqns. (3.3) to (3.6) through the dual variables λ_1^m, λ_{2j}^m, λ_{3j}^m and λ_{4j}, respectively as follows:

$$
\begin{aligned}
\alpha(\ldots) = \sum_m \Big[\sum_{i\in S} \{a_i {P_{si}^m}^2 + b_i P_{si}^m + c_i\} - \lambda_1^m \Big(\sum_{i\in S} P_{si}^m + \sum_{j\in H} P_{Hj}^m \\
- P_L^m - P_D^m \Big) + \sum_{j\in H} \lambda_{2j}^m \{ P_{Hj}^m - h_j(1+0.5c_j(2X_j^m + J_j^m - Q_j^m))\} \\
(Q_j^m - \rho_j) + \sum_j \lambda_{3j}^m (X_j^{m+1} - X_j^m - J_j^m + Q_j^m) \Big] + \sum_j \lambda_{4j} (X_j^{M+1} - X_j^1 \\
- \sum_m J_j^m + \sum_m Q_j^m \Big) \qquad (5.12)
\end{aligned}
$$

During each subinterval, the control variables are the water discharges through the turbines of the hydroelectric plants. The hydroelectric storages and the power generation at the end of a subinterval are obtained from eqns. (3.3) and (3.4), respectively. Irrespective of the actual hydrogenerations, the thermal generations satisfy eqn. (5.13) for minimal fuel cost and power transfer eqn. (3.6). These are taken as dependent variables.

The dual variables λ_1^m, λ_{2j}^m and λ_{3j}^m are obtained by equating the partial derivatives of the Lagrangian function with respect to the dependent variables to zero.

$$\left(\frac{\partial\alpha(\ldots)}{\partial P_{si}^m}\right)_{i\epsilon s} = b_i+2a_i\ P_{si}^m - \lambda_1^m(1-\frac{\partial P_L^m}{\partial P_{si}^m}) = 0 \qquad (5.13)$$

$$\left(\frac{\partial\alpha(\ldots)}{\partial P_{Hj}^m}\right)_{j\epsilon H} = \lambda_{2j}^m - \lambda_1^m\ (1-\frac{\partial P_L^m}{\partial P_{Hj}^m}) = 0 \qquad (5.14)$$

$$\left(\frac{\partial\alpha(\ldots)}{\partial X_j^m}\right)_{\substack{j\epsilon H\\ m\neq 1}} = \lambda_{3j}^{m-1} - \lambda_{3j}^m - \lambda_{2j}^m\ h_j\ c_j(Q_j^m - \rho_j) = 0 \qquad (5.15)$$

$\lambda_{3j}^1 = 0$, since the equations associated with this set of dual variables are redundant.

The gradient vector is

$$\left(\frac{\partial\alpha(\ldots)}{\partial Q_j^m}\right)_{j\epsilon H} = \lambda_{3j}^m + \lambda_{4j} - \lambda_{3j}^m\ h_j\ \{1+0.5c_j(2X_j^m + J_j^m - 2\ Q_j^m + \rho_j)\} \qquad (5.16)$$

The dual variables λ_{4j} are to be adjusted to maximise the Lagrangian function of eqn. (5.12) under the constraints of other optimality conditions. The corresponding gradient vector is

$$\left(\frac{\partial \alpha(\ldots)}{\partial \lambda_{4j}}\right)_{j\varepsilon H} = X_j^{M+1} - X_j^1 - \sum_m J_j^m + \sum_m Q_j^m \tag{5.17}$$

The upper and lower limits on the control variables are taken care of as described in Chapter 3.

5.4 COMPUTATIONAL ALGORITHM

The computational algorithm is based upon Fletcher's technique using principle of decomposition. The computational algorithm is illustrated in flowchart shown in Fig. 5.1. It starts with the assumption of a set of λ_{4j} and a feasible set of control variables Q_j^m for all hydroplants.

5.5 SAMPLE SYSTEM STUDY

A long range problem with two thermal generations and two hydro-electric plants as outlined in section 3.6 of Chapter 3 is considered to study the implementation of Fletcher's technique. The relevant deterministic data of the problem is given below:

$$\frac{\partial F_1}{\partial P_{s1}} = 0.80 + 0.04\ P_{s1}$$

$$\frac{\partial F_2}{\partial P_{s2}} = 0.78 + 0.06\ P_{s2}$$

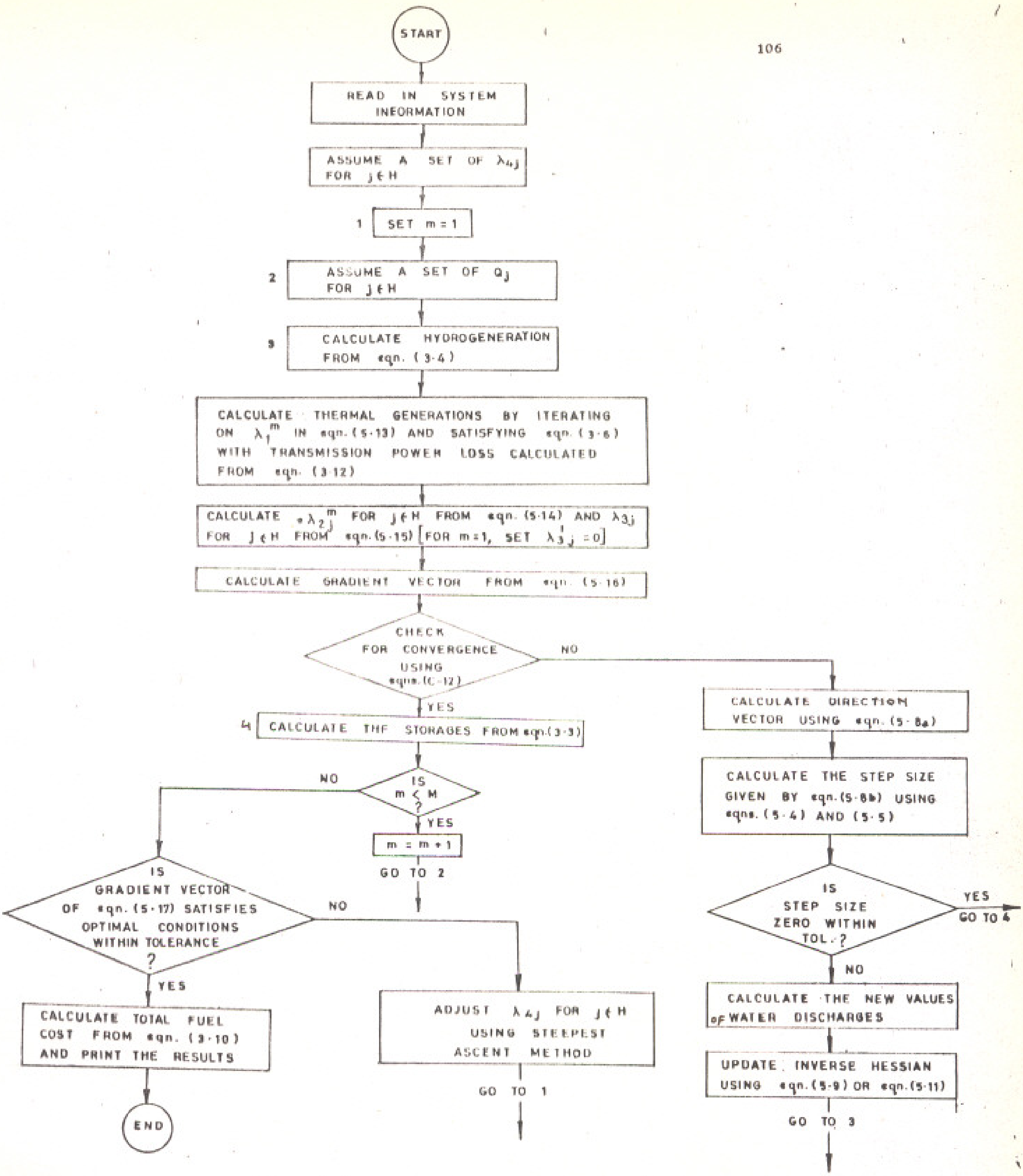

Fig. 5.1 Flow chart for the optimal scheduling of hydrothermal systems using Fletcher's technique.

Data in per unit for the two hydroplants are presented in Table 5.1. The water inflows for the six subintervals are given in Table 5.2.

The load demand is 8.0 p.u. for each subinterval and the loss formula matrix is

$$\begin{bmatrix} 0.05 & -0.02 & 0.01 & 0.00 \\ -0.02 & 0.06 & -0.02 & 0.01 \\ 0.01 & -0.02 & 0.04 & 0.00 \\ 0.00 & 0.01 & 0.00 & 0.02 \end{bmatrix}$$

Table 5.1: HYDROPLANT DATA IN PER UNIT

	Hydroplant	
	1	2
h, c, ρ	1.0, 0.1 and 0.1 respectively	0.6, 0.12 and 0.05 respectively
Initial Storages	10	12
Final Storages	10	12
Minimum and maximum allowable discharges	0.5, 3.5 respectively	0.5, 3.5 respectively

Table 5.2: WATER INFLOWS IN PER UNIT

Hydro plant / Sub-Interval m =	1	2	3	4	5	6
1	0.0	0.6	1.2	1.2	1.2	1.8
2	0.0	0.0	0.0	1.5	3.0	4.5

With the initial values of variables assumed as given in Table 3.3, the optimal values obtained through the Fletcher's technique of Section 5.2 are given in Table 5.3.

Table 5.3: OPTIMISED VALUES IN PER UNIT USING FLETCHER'S TECHNIQUE

m	Water Discharges per unit		Hydrogenerations per unit		Thermal generations per unit	
	Q_1^m	Q_2^m	P_{H1}^m	P_{H2}^m	P_{s1}^m	P_{s2}^m
1	0.7456	1.1575	1.2672	1.5753	3.0720	2.8256
2	0.8048	1.2138	1.3498	1.5560	3.0161	2.8040
3	0.9554	1.2083	1.6400	1.4476	2.8459	2.7539
4	1.0659	1.2250	1.8701	1.4290	2.6790	2.6807
5	1.0806	1.9573	1.9109	2.4100	2.1359	2.1467
6	1.3877	2.2531	2.5436	3.0447	1.3858	1.6392

Fuel cost = 25.6343 per unit

5.6 CONCLUSIONS

To the best of the author's knowledge, Fletcher's technique has been applied for the first time to optimal power dispatch of hydrothermal systems. A common feature is the elimination of the uni-dimensional search at each stage which makes it more attractive due to simplicity, efficiency and ease of implementation. The algorithm has been programmed and applied to a hydrothermal system comprising 2-hydro and 2-thermal systems. It has been found that the number of iterations for the solution of primal problem are much less as compared to the conjugate gradient method but at the expense of accuracy as the solution converges to suboptimal one compared to conjugate gradient method because of large value of tolerance for the dual problem.

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CHAPTER - 6

CONCLUSIONS

6.1 INTRODUCTION

This chapter presents the salient contributions of the thesis as well as brief statement concerning a few recommendations for further research. The original contributions are listed and discussed.

6.2 RESEARCH CONCLUSIONS

This thesis has investigated in terms of probability and statistics, the feasibility of quantitative representation of inaccuracies and uncertainties of the input data for the optimal power dispatch problem. The following contributions result from this research:

1. Stochastic optimal thermal power dispatch problem has been formulated and the solution of the new formulation has been obtained by direct extension of the existing methods of analysis. The possible economic significance of the stochastic formulation has been illustrated by carrying out studies on a large sample system. The research stresses that the deterministic schedules may result in the lowest expected cost but the cost associated with such schedules has relatively large variance which can be interpreted as risk measure. Deterministic schedules, therefore, render the solution non-optimal

to a power system analyst who would like to avoid the risk element. The new formulation proposes that to generate a robust optimal thermal power dispatch procedure, the power control engineer should take into account of a second best but deterministic equivalent approach which is to minimise simultaneously the expected fuel cost and its sensitivity to deviations, thereby imposing additional penalty costs proportional to the expected value of such deviations. The proposed stochastic model results in an enhancement of the optimal value of the expected cost over the classical deterministic dispatch model. The resulting increase in the value of the cost has been interpreted as the risk premium which a power control engineer is expected to pay in making risk aversion decisions for accounting of the inaccuracies in the input data. Results of the sample system demonstrate the suitability of this method for application to a actual power system.

2. Hydrothermal scheduling problem has been analysed using a stochastic decision model. The uncertainties present in the water inflows, system load demand and operating cost coefficients have been effectively accounted for by characterising the problem variables stochastic rather than deterministic. A deterministic equivalent of the stochastic formulation is obtained. The deterministic equivalent problem has been solved using duality decomposition in order to obtain simpler subproblems requiring lesser storage and computation time. In each sub-interval, the thermal subproblem has been solved by classic dispatch and

hydrosubproblem has been solved using conjugate gradient method. The practical viability of the formulation has been demonstrated through a sample system study.

3. The generality, effectiveness and practical applicability of the stochastic formulation has been further demonstrated by applying it to the scheduling of cascaded hydrothermal systems. Inaccuracies in the operating cost coefficients have been tackled for the first time for solving the cascaded problem.

4. Finally, hydrothermal scheduling problem has been solved using Fletcher's modified variable metric method which avoids an inverse interpolation in performing the linear search and implements an 'acceptable point' linear search procedure. It has been found that the number of iterations for the solution of the primal problem are much less as compared to conjugate gradient method but the solution converges to suboptimal point because of increased value of tolerance for the solution of dual problem. Admittedly, the algorithm in present form cannot be implemented in view of convergence to suboptimal solution but further investigations are warranted to overcome this difficulty.

6.3 FUTURE LINE OF RESEARCH

Additional research topics include:

1. The question of optimum power system operation with respect to multiobjective functionals lacks an answer so far. It would be worthwhile to investigate these problems using

available theoretical results. The multiobjective formulation is particularly amenable to sensitivity analysis. A sensitivity index as a function of the relative size of the perturbations in the solution due to variations in the parameters could be specified. Once the index is specified, it can be included as an additional objective to be minimised in the multiobjective system.

2. Unit commitment forms an important part of operation planning of power systems. In the economic operation problem, reasonable schedule for generating equipment and economic power dispatch can be obtained as part of an integrated operation schedule by including uncertainties in operating cost coefficients as well as in start up costs of each unit.

3. A more comprehensive analysis may be carried out by incorporating risk aversion in optimal power flow studies.

4. Developing efficient simulators and on-line controllers for implementing the optimum strategies is another area of endeavour.

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APPENDIX - A

EFFECTS OF ERRORS IN ECONOMIC DISPATCHING OF POWER SYSTEMS

An analysis of the effects of errors in the economic dispatching is important in understanding and choosing the accuracy requirements of the components of a dispatching system.

This appendix contains a derivation for calculation of change in fuel cost for various values of load demand and derivation of expression for hourly loss in operating economy resulting from error in incremental cost representation.

A.1 CALCULATION OF ΔF_T FOR VARIOUS VALUES OF P_D

Let Fig. A.1 represent the exact incremental cost-data for a given incremental cost and the corresponding value of P_D. Let the values of P_{s1} and P_{s2} obtained with the exact incremental data be denoted by P'_{s1} and P'_{s2}.

If the modified incremental-cost data is used, different values of P_{s1} and P_{s2} will be obtained for the same value of P_D. Let these values of P_{s1} and P_{s2} be denoted by P''_{s1} and P''_{s2}.

In going from P'_{s2} to P''_{s2} the fuel input to unit is increased by

$$F'_2 = a_2 P'^2_{s2} + b_2 P'_{s2}$$

$$F''_2 = a_2 P''^2_{s2} + b_2 P''_{s2}$$

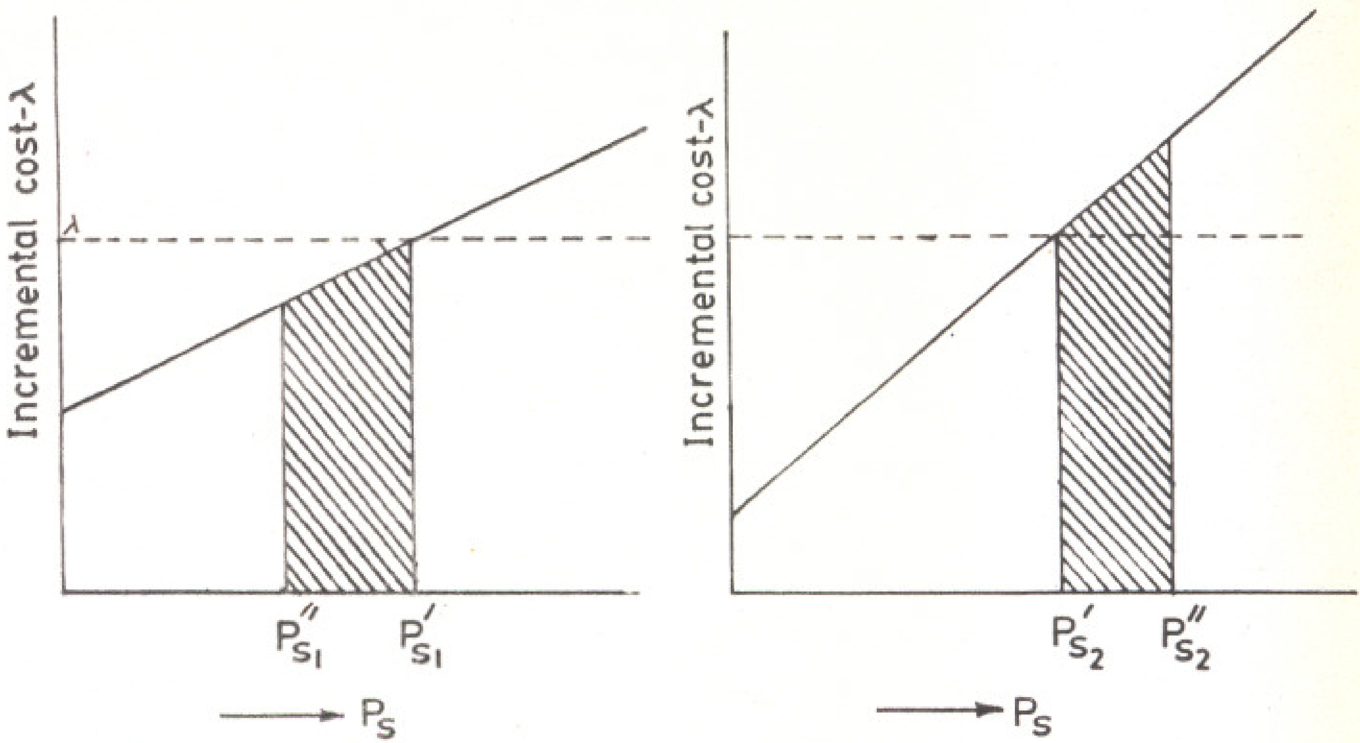

Fig. A.1 Incremental cost data

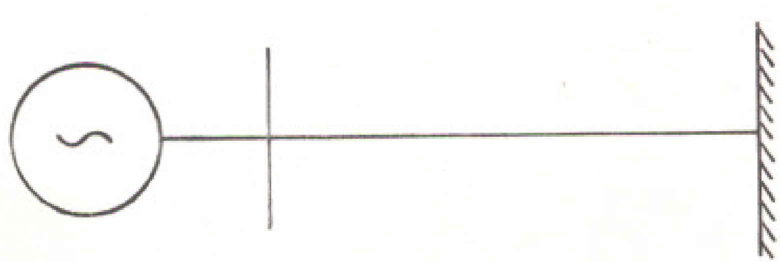

Fig B.1 Simplified system representation

$$\Delta F_2 = F''_2 - F'_2 = a_2(P''^2_{s2} - P'^2_{s2}) + b_2(P''_{s2} - P'_{s2})$$

or $$\Delta F_2 = (P''_{s2} - P'_{s2})\left\{ \frac{2a_2P''_{s2}+b_2+2a_2P'_{s2}+b_2}{2} \right\}$$

$$= \left[\frac{\left(\frac{dF_2}{dP_{s2}}\right)'' + \left(\frac{dF_2}{dP_{s2}}\right)'}{2} \right] \Delta P_{s2}$$

where $\Delta P_{s2} = P''_{s2} - P'_{s2}$ = change in generation on unit 2.

$$\left(\frac{dF_2}{dP_{s2}}\right)'' = \frac{dF_2}{dP_{s2}} \text{ for } P_{s2} = P''_{s2}$$

$$\left(\frac{dF_2}{dP_{s2}}\right)' = \frac{dF_2}{dP_{s2}} \text{ for } P_{s2} = P'_{s2}$$

The fuel input to unit 1, however, is decreased by

$$\Delta F_1 = \left[\frac{\left(\frac{dF_1}{dP_{s1}}\right)'' + \left(\frac{dF_1}{dP_{s1}}\right)'}{2} \right] \Delta P_{s1}$$

Since the load dropped by one unit must be picked up by the other, the change in P_{s1} must be equal and opposite to the change in P_{s2}. Thus

$$\Delta P = -\Delta P_{s1} = \Delta P_{s2}$$

Therefore, the change in total fuel input is given by

$$\Delta F_T = \Delta F_1 + \Delta F_2$$

$$= \frac{\Delta P}{2} \{ [(\frac{dF_2}{dP_{s2}})'' + (\frac{dF_2}{dPs_2})'] - [(\frac{dF_1}{dP_{s1}})'' + (\frac{dF_1}{dP_{s1}})']\}$$

but $(\frac{dF_1}{dP_{s1}})' = (\frac{dF_2}{dP_{s2}})'$

Therefore,

$$\Delta F_T = [(\frac{dF_2}{dP_{s2}})'' - (\frac{dF_1}{dP_{s1}})'] \frac{\Delta P}{2} \qquad (A.1)$$

Using eqn. (A.1), ΔF_T may be quickly and accurately calculated.

A.2 DERIVATION OF EXPRESSION FOR HOURLY LOSS IN OPERATING ECONOMY RESULTING FROM ERROR IN INCREMENTAL COST REPRESENTATION

It is required to calculate the loss of operating economy involved when the incremental operating cost associated with two units is modified.

Let $\frac{dF_1}{dP_{s1}} = 2a_1 P_{s1} + b_1$; and

$$\frac{dF_2}{dP_{s2}} = 2a_2 P_{s2} + b_2$$

Assume that $\frac{dF_1}{dP_{s1}}$ is multiplied by $(1+\varepsilon)$ and that $\frac{dF_2}{dP_{s2}}$ is multiplied by $(1-\varepsilon)$. The new schedule obtained is given by solution of equations:

$$(1+\varepsilon)\left[2a_1 P''_{s1} + b_1\right] = (1-\varepsilon)\left[2a_2 P''_{s2} + b_2\right]$$

$$P''_{s1} + P''_{s2} = P_D$$

giving

$$P''_{s1} = \frac{2a_2 P_D(1-\varepsilon) + b_2(1-\varepsilon) - b_1(1+\varepsilon)}{2a_1(1+\varepsilon) + 2a_2(1-\varepsilon)}$$

$$P''_{s2} = \frac{2a_1 P_D(1+\varepsilon) + b_1(1+\varepsilon) - b_2(1-\varepsilon)}{2a_2(1-\varepsilon) + 2a_1(1+\varepsilon)}$$

Taking the case of two identical units

$$2a_1 = 2a_2 = a$$

$$b_1 = b_2 = b$$

$$P''_{s1} = \frac{P_D}{2}(1-\varepsilon) - \varepsilon\frac{b}{a}$$

The shift in loading for each of the two identical units may be computed from the expression

$$\Delta P = \frac{P_D}{2} - P''_{s1}$$

$$= \frac{P_D}{2} - \left[\frac{P_D}{2}(1-\varepsilon) - \varepsilon\frac{b}{a}\right]$$

$$= \left(\frac{\frac{a}{2}P_D + b}{a}\right)\varepsilon \qquad \text{(A.2)}$$

The hourly loss in fuel economy is given by eqn. (A.1) where

$$\left(\frac{dF_2}{dP_{s2}}\right)'' = a\left(\frac{P_D}{2} + \Delta P\right) + b\,, \quad \left(\frac{dF_1}{dP_{s1}}\right)'' = a\left(\frac{P_D}{2} - \Delta P\right) + b$$

$$\Delta F_T = 2a \ \Delta P \frac{\Delta P}{2}$$

$$\Delta F_T = a \, \Delta P^2 \qquad (A.3)$$

Substituting eqn.(A.2) into eqn. (A.3)

$$\Delta F_T = \left[\frac{a}{2} P_D + b \right]^2 \frac{\varepsilon^2}{a} \qquad (A.7)$$

For the case of two identical units, with no transmission losses, the incremental cost of received power is

$$a P_{s1} + b = \lambda \qquad (A.4)$$

$$a P_{e2} + b = \lambda \qquad (A.5)$$

Adding eqns. (A.4) and (A.5)

$$= \frac{a}{2} P_D + b \qquad (A.6)$$

Substituting eqn. (A.6) into eqn. (A.7), we obtain

$$\Delta F_T = \lambda^2 \frac{\varepsilon^2}{a} \qquad (A.8)$$

It is seen from eqn. (A.8) that the deviation of fuel economy varies as the square of error ε.

Equation (A.7) expresses the loss in operating economy in terms of the total generation as

$$\Delta F_T = \left(\frac{a}{2} P_D + b \right)^2 \frac{\varepsilon^2}{a}$$

Equations (A.7) and (A.8) are applicable only when both units are unconstrained by minimum or maximum generation limitations.

APPENDIX - B

EVALUATION OF COEFFICIENT OF VARIATION OF GENERATOR OUTPUT

This appendix describes a possible method of Monte Carlo simulation to obtain sample estimate of the expected value and variance of generator output variable via incremental production cost model incorporating random coefficients. The ratio betwen standard deviation and mean of the output variable is computed and interpreted in terms of coefficient of variation of the generator.

B.1 EVALUATION OF COEFFICIENT OF VARIATION

Consider an existing power system with (n-1) thermal generating units operating at equal incremental production costs to supply a system load. Knowing the value of incremental production cost, the power outputs of individual generators can be calculated. Fig.B.1 shows an additional nth thermal generating unit brought on to this system to pick up increasing load on the system. If the incremental production cost equation of this generator is stochastic and incorporates random coefficients, then for a particular value of system λ, the generator output is random and is given by

$$P = \frac{\lambda - b}{2a} \qquad (B.1)$$

The Monte carlo method essentially consists of the generation of a large number of repetitive solutions of the incremental production cost model from which sample statistics of the generator output can be calculated using eqn. (B.1). Random coefficients are introduced into the incremental production cost equation in a manner which conceptually simulated the way the uncertainty enters into these coefficients. These coefficients being estimated from experimental data are assumed to be normally distributed random variables. If only two parameters, such as mean and standard deviation, are known for normal distribution, it represents the maximum known information concerning the random variable. Each of the values of the random variables employed were computed by adding the generated [146] random variable to a deterministic quantity as for example:

$$a = \bar{a} + \sigma_{\bar{a}} \cdot \varepsilon_N$$

$$b = \bar{b} + \sigma_{\bar{b}} \cdot \varepsilon_N$$

Addition of the random error (ε_N) having the desired distribution to the deterministic cost coefficients ($\bar{a}$ and $\bar{b}$) yielded random variables that were used in simulation. $\sigma_{\bar{a}}$ and $\sigma_{\bar{b}}$ are ensemble standard deviations of normally distributed random variables. Characteristics of the random variables can be controlled through the values of $\sigma_{\bar{a}}$ and $\sigma_{\bar{b}}$. Table B.1 gives the values of coefficient of variation estimated from sample mean and sample standard deviation of a sample of 100 values with 10 percent coefficient of variation for 'a' and 'b' coefficients of production cost curve. Studies with different values of λ in eqn. (B.1) for any generator for which its output lies between minimum and maximum generation limits, did not give much variation in its coefficient of variation. For example, in plant 8 of the large system [2] studied, values of λ lying in the range 1.7 to 2.3 changed the coefficient of variation from 14 percent to 12.5 percent.

Table B.1: COEFFICIENT OF VARIATION OF GENERATOR OUTPUT

Plant Number	Value of	Estimated value of coefficient of variation (%).
1	2.40	20
2	1.72	17
6	2.10	11
7	2.10	13
8	2.00	13

APPENDIX - C

PENALTY COSTS, TRANSMISSION LOSS, PROBABILITY PROPERTIES OF HYDROELECTRIC GENERATION AND OPTIMALITY CONDITIONS

This appendix contains derivation for penalty costs, expected transmission loss and probability properties of hydroelectric generation. It also specifies optimality conditions for convergence of NLP problem.

C.1 EVALUATION OF PENALTY COSTS

Thermal Power Plants:

The load demand constraint as given by eqn. (2.17a) is

$$\sum_i \bar{P}_{si} = \bar{P}_D + \bar{P}_L$$

Expected value of the square of any possible deviations of the random variables P_{si} from their respective expected values is

$$E\left[(\bar{P}_D + \bar{P}_L - \sum_i P_{si})^2\right] \tag{C.1a}$$

Assuming random variables P_{si} to be statistically independent, eqn. (C.1a) simplifies to

$$\sum_i \text{var}(P_{si})$$

Therefore, the penalty cost is

$$p.\left[\sum_i \text{var}(P_{si})\right]$$

Hydro Power Plants:

The load demand constraint as given by eqn. (3.6) is

$$\sum_{i\varepsilon s} \bar{P}^m_{si} + \sum_{j\varepsilon H} \bar{P}^m_{Hj} = \bar{P}^m_D + \bar{P}^m_L$$

Expected value of the square of any possible deviations of the random variables P^m_{si} and P^m_{Hj} from their respective expected values is

$$E\left[(\bar{P}^m_D + \bar{P}^m_L - \sum_{i\varepsilon s} P^m_{si} - \sum_{j\varepsilon H} P_{Hj})^2\right] \tag{C.1b}$$

Assuming random variables P^m_{si} to be statistically independent and random variables P^m_{Hj} and P^m_{Hv} to be statistically correlated, eqn. (C.1b) simplifies to

$$\sum_{i\varepsilon s} \text{var}\, P^m_{si} + \sum_{j\varepsilon H} \sum_{v\varepsilon H} \text{Cov}(P^m_{Hj}, P^m_{Hv})$$

where $E[P^m_{Hj} \cdot P^m_{Hv}] = \bar{P}^m_{Hj} \cdot \bar{P}^m_{Hv} + \text{cov}(P^m_{Hj}, P^m_{Hv})$

Therefore, the penalty cost is

$$p.\left[\sum_{i\varepsilon s} \text{var}\,(P^m_{si}) + \sum_{j\varepsilon H} \sum_{v\varepsilon H} \text{Cov}(P^m_{Hj}, P^m_{Hv})\right] \tag{C.2}$$

C.2 EXPECTED TRANSMISSION LOSS MODEL

Thermal Power Plants:

The random transmission loss as expressed in terms of B-coefficients is given by

$$P_L = \sum_{i\epsilon s} \sum_{j\epsilon s} P_{si} B_{ij} P_{sj}$$

with random variables P_{si} statistically independent, the expected transmission loss is,

$$E[P_L] = \sum_{i\epsilon s} \sum_{j\epsilon s} \bar{P}_{si} B_{ij} \bar{P}_{sj} + \sum_{i\epsilon s} B_{ii} \, var(P_{si}) \tag{C.3a}$$

Hydro Power Plants:

If n is the number of thermal plants and (β-n) is the number of hydroplant, then the random transmission loss as expressed in terms of B-coefficients is given by

$$P_L^m = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{si}^m B_{ij} P_{sj}^m + \sum_{u=n+1}^{\beta} \sum_{v=n+1}^{\beta} P_{Hu}^m B_{uv} P_{Hv}^m$$

$$+ 2 \sum_{j=1}^{n} \sum_{v=n+1}^{\beta} P_{sj}^m B_{jv} P_{Hv}^m \tag{C.3b}$$

With random variables P_{si}^m statistically independent and random variables P_{Hj}^m, P_{Hv}^m statistically correlated, the expected transmission loss is

$$E[P_L] = \sum_{i=1}^{n} \sum_{j=1}^{n} \bar{P}_{si}^m B_{ij} \bar{P}_{sj}^m + \sum_{u=n+1}^{\beta} \sum_{v=n+1}^{\beta} \bar{P}_{Hu}^m B_{uv} \bar{P}_{Hv}^m$$

$$+ 2 \sum_{j=1}^{n} \sum_{v=n+1}^{\beta} \bar{P}_{sj}^m B_{jv} \bar{P}_{Hv}^m + \sum_{i\epsilon s} B_{ii} \, var \, P_{si}^m +$$

$$+ \sum_{j=n+1}^{p} \sum_{v=n+1}^{p} B_{jv} \operatorname{Cov}(P_{Hj}^{m}, P_{Hv}^{m}) \tag{C.4}$$

C.3 PROBABILITY PROPERTIES OF HYDROELECTRIC GENERATION

Water inflows into the reservoirs of various hydroelectric plants are assumed to be statistically correlated during the same sub-interval but independent at different subintervals.

Storage continuity equation gives

$$X_j^{m+1} = X_j^m + J_j^m - Q_j^m \tag{C.5}$$

Expected value of eqn. (C.5)

$$\bar{X}_j^{m+1} = \bar{X}_j^m + \bar{J}_j^m - Q_j^m \tag{C.6}$$

Corresponding variances are

$$\operatorname{var}(X_j^{m+1}) = \operatorname{var}(X_j^m) + \operatorname{var}(J_j^m) \tag{C.7}$$

Knowing the variance of X_j^m and variance of J_j^m, variance of X_j^{m+1} can be found from eqn. (C.7)

Similarly,

$$\operatorname{Cov}(X_j^{m+1}, X_v^{m+1}) = \operatorname{Cov}(X_j^m, X_v^m) + \operatorname{Cov}(J_j^m, J_v^m) \tag{C.8}$$

The expression for hydropower is given by

$$(P_{Hj}^m)_{j\varepsilon H} = h_j \{1+0.5c_j(2X_j^m+J_j^m-Q_j^m)\} (Q_j^m - \rho_j)$$

$P_{Hj}^m (j\varepsilon H)$ is random because of randomness associated with variables X_j^m and J_j^m.

Expected value of hydropower is given as follows:

$$\bar{P}_{Hj}^m = E[P_{Hj}^m] = h_j\{1+0.5c_j(2\bar{X}_j^m + \bar{J}_j^m - Q_j^m)\}(Q_j^m - \rho_j) \tag{C.9}$$

$$\mathrm{Var}(P_{Hj}^m)_{j\varepsilon H} = E[(P_{Hj}^m - \bar{P}_{Hj}^m)^2]$$

$$= h_j^2\ (Q_j^m - \rho_j)^2 c_j^2\{\mathrm{var}(X_j^m) + 0.25\ \mathrm{var}(J_j^m)\} \tag{C.10}$$

$$\{\mathrm{cov}\ (P_{Hj}^m,\ P_{Hv}^m)\}_{\substack{j\varepsilon H\\ v\varepsilon H}} = \rho(P_{Hj}^m,\ P_{Hv}^m)\{\mathrm{var}\ (P_{Hj}^m).\mathrm{var}(P_{Hv}^m)\}^{\frac{1}{2}}$$

where $\rho(P_{Hj}^m,\ P_{Hv}^m)$ is the correlation coefficient.

Taking $\rho(P_{Hj}^m,\ P_{Hv}^m) = 1$

$$\mathrm{Cov}(P_{Hj}^m,\ P_{Hv}^m) = h_j\ h_v(Q_j^m - \rho_j)(Q_v^m - \rho_v)c_j\ c_v\{\mathrm{cov}(X_j^m, X_v^m)$$

$$+0.25\ \mathrm{cov}(J_j^m,\ J_v^m)\} \tag{C.11}$$

$$\{\mathrm{Cov}(X_j^{m+1}, X_v^{m+1})\}_{\substack{j\varepsilon H\\ v\varepsilon H}} = E[(X_j^{m+1} - \bar{X}_j^{m+1})(X_v^{m+1} - \bar{X}_v^{m+1})]$$

where X_j^{m+1}, is the random value of the storage of the j-th plant at the beginning of (m+1)th subinterval.

Now $X_j^{m+1} = X_j^m + J_j^m - Q_j^m$

$$X_v^{m+1} = X_v^m + J_v^m - Q_v^m$$

$$\mathrm{Cov}(X_j^{m+1}, X_v^{m+1}) = \mathrm{Cov}(X_j^m, X_v^m) + \mathrm{Cov}(J_j^m, J_v^m)$$

Because

$$E\left[(X_j^m - \bar{X}_j^m)(J_v^m - \bar{J}_v^m)\right] = 0$$

$$E\left[(J_j^m - \bar{J}_j^m)(X_v^m - \bar{X}_v^m)\right] = 0$$

C.4 OPTIMALITY CONDITIONS

Optimal conditions for an objective function $f(x)$ to be minimum under the constraints $X_{imin} \leq X_i \leq X_{imax}$ are

$$\frac{df(x)}{dX_i} = 0 \quad \text{if} \quad X_{imin} \leq X_i \leq X_{imax}$$

$$\frac{df(x)}{dX_i} \geq 0 \qquad \text{if } X_i = X_{imin} \tag{C.12}$$

$$\frac{df(x)}{dX_i} \leq 0 \qquad \text{if } X_i = X_{imax}$$

APPENDIX - D

PROBABILITY PROPERTIES OF CASCADED HYDROPLANTS

This appendix contains probability properties of hydroelectric systems with cascaded plants.

Water continuity equation gives

$$X_j^{m+1} = X_j^m + J_j^m - Q_j^m \qquad (D.1)$$

$$X_{jo}^{m+1} = X_{jo}^m + J_{jo}^m + Q_j^{m-\tau} - Q_{jo}^m \qquad (D.2)$$

Taking expectation of eqns. (D.1) and (D.2), we get

$$\bar{X}_j^{m+1} = \bar{X}_j^m + \bar{J}_j^m - Q_j^m \qquad (D.3)$$

$$\bar{X}_{jo}^{m+1} = \bar{X}_{jo}^m + \bar{J}_j^m + Q_j^{m-\tau} - Q_{jo}^m \qquad (D.4)$$

Corresponding variances are

$$\text{var}(X_j^{m+1}) = \text{var}(X_j^m) + \text{var}(J_j^m); \quad jUj_o \in H \qquad (D.5)$$

Knowing the variance of X_j^m and variance of J_j^m, variance of X_j^{m+1} can be found from eqn. (D.5).

Similarly,

$$\text{Cov}(X_j^{m+1}, X_{jo}^{m+1}) = \text{Cov}(X_j^m, X_{jo}^m) + \text{Cov}(J_j^m, J_{jo}^m) \qquad (D.6)$$

The expressions for hydropowers are given by

$$(P_{Hj}^m)_{j\epsilon H} = h_j \{1+0.5c_j(2X_j^m + J_j^m - Q_j^m)\} (Q_j^m - \rho_j) \qquad (D.7)$$

$$(P^m_{Hjo})_{jo\varepsilon H} = h_{jo}\{1+0.5c_{jo}(2X^m_{jo}+J^m_{jo}+Q^{m-1}_{jo} - Q^m_{jo})\}\ (Q^m_{jo} - \ell_{jo}) \tag{D.8}$$

$P_{hj}(jUj_o\varepsilon H)$ is random because of randomness associated with variables X^m_j and J^m_j.

Expected values of hydropowers of eqns. (D.7) and (D.8) are given as follows:

$$\bar{P}^m_{Hj} = E\,[\,P^m_{Hj}\,]$$

$$\bar{P}^m_{Hj} = h_j\{1+0.5c_j(2\bar{X}^m_j + \bar{J}^m_j - Q^m_j)\}\ (Q^m_j - \ell_j) \tag{D.9}$$

Similarly

$$\bar{P}_{Hjo} = h_{jo}\{1+0.5c_{jo}(2\bar{X}^m_{jo}+\bar{J}^m_j + Q^{m-1}_j - Q^m_{jo})\}(Q^m_{jo} - \ell_{jo}) \tag{D.10}$$

$$\{Var\ (P^m_{Hj})\}_{jUjo\varepsilon H} = h^2_j\ (Q^m_j - \ell_j)^2 c^2_j\ \{var(X^m_j)+0.25var(J^m_j)\} \tag{D.11}$$

$$\{cov(P^m_{Hj}, P^m_{Hjo})\}_{jUjo\varepsilon H} = h_j\ h_{jo}(Q^m_j - \ell_j)\ (Q^m_{jo} - \ell_{jo})c_j\ c_{jo}\{cov(X^m_j, X^m_{jo}) + 0.25\ cov(J^m_j,\ J^m_{jo})\} \tag{D.12}$$

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i) "Economic Dispatch of Thermal Generation" (with Professor D.P.Kothari and Prof.P.V.Gupta), Proc. Intn. AMSE Conf., Paris, July 1-3, 1982, AMSE Press, Vol.8, pp.20.

ii) "Economic Thermal Power Dispatch" (with Prof.D.P. Kothari and Prof.P.V.Gupta), I.E.(I), Vol.64, pt.EL2, October 1983, pp.125-132.

iii) "Optimal Hydrothermal Generation Scheduling" (with Prof. D.P.Kothari and Prof.P.V.Gupta), IIIrd National Power Systems Conference, Regional Engineering College, Warangal, Sept.8-10, 1984.

iv) "Robust Hydrothermal Economic Power Dispatch" (with Prof. D.P.Kothari and Prof.P.V.Gupta), IVth National Power Systems Conference, Institute of Technology, Varanasi, Feb.15-17, 1986.

v) "Stochastic Optimal Hydrothermal Power Dispatch" (with Prof. D.P.Kothari and Prof.P.V.Gupta), IFAC Symposium on Automation and Instrumentation for Power Plants, Bangalore (India), Dec.15-17, 1986.

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