Design and Simulation of Koch Fractal Antenna Array for Mobile Communications

A Thesis
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In
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<td>IFS</td>
<td>Iterated Function System</td>
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<tr>
<td>FBR</td>
<td>Front to-Back Ratio</td>
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<td>VSWR</td>
<td>Voltage Wave Standing Ratio</td>
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<td>IE</td>
<td>Integral Equation Method</td>
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<td>MOM</td>
<td>Methods of Moments</td>
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<td>FBW</td>
<td>Frequency Bandwidth</td>
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<td>GSM</td>
<td>Global System for Mobile Communication</td>
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<td>GPS</td>
<td>Global Positioning System</td>
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<td>MIMO</td>
<td>Multi-input Multi-output</td>
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Abstract

With the advance of wireless communication systems and increasing importance of other wireless applications, wideband and low profile antennas are in great demand or both commercial and military applications. Multi-band and wideband antennas are also desirable in personal communication systems, small satellite communication terminals, and other wireless applications. Wideband antennas also find applications in Unmanned Aerial Vehicles (UAVs), Synthetic Aperture Radar (SAR), and Ground Moving Target Indicators (GMTI). Some of these applications also require that an antenna be embedded into the airframe structure traditionally, a wideband antenna in the low frequency wireless bands can only be achieved with heavily loaded wire an antenna, which usually means different antennas, are needed for different frequency bands.

Recent progress in the study of fractal antennas suggests some attractive solutions for using a single small antenna operating in several frequency bands. The term fractal, which means broken or irregular fragments, was originally coined by Mandelbrot to describe a family of complex shapes that possess an inherent self similarity in their geometrical structure. Fractals found widespread use in many branches of science and engineering in a relatively short time. Electromagnetism, and in particular antenna design has also benefited from these concepts. Applying fractals to antenna elements allows for smaller, resonant antennas that are multiband/broadband and may be optimized for gain.

In this thesis three elements Koch antenna array is proposed. the performance of three Element Koch array is simulated and results obtained are compared with single Koch antenna. It is found from the analysis that the Koch array improves the gain, directivity, Bandwidth and input impedance. The frequencies used in simulation are of mobile communication viz GSM 900 and GSM 1800 MHz.
CHAPTER 1
INTRODUCTION

1.1 INTRODUCTION

There has been an ever growing demand, in both the military as well as the commercial sectors, for antenna design that possesses the following highly desirable attributes:

i) Compact size
ii) Low profile
iii) Conformal
iv) Multi-band or broadband

There are a variety of approaches that have been developed over that year, which can be utilized to achieve one or more of these design objectives. The term fractal, which means broken or irregular fragments, was originally coined by Mandelbrot to describe a family of complex shapes that possess an inherent self similarity in their geometrical structure. The original inspiration for the development of fractal geometry came largely from in depth study of the patterns of nature. For instance, fractals have been successfully used to model such complex natural objects as galaxies; cloud boundaries, mountain ranges, coastlines, snowflakes, trees. Leaves, fern, and much more a wide variety of applications for fractals continue to be found in many branches of science and engineering. This geometry, which has been used to model complex objects found in nature such as clouds and coastlines, has space-filling properties that can be utilized to miniaturize antennas [25]. One such area is fractal electromagnetic theory for the purpose of investigating a new class of radiation, propagation, and scattering problems. One of the most promising areas of fractal electrodynamics research is in its application to antenna theory and design. Modern telecommunication systems require antennas with wider bandwidths and Smaller Dimensions than conventionally possible. This has initiated antenna research in various directions, one of which is by using fractal shaped antenna elements. In
recent years several fractal geometries have been introduced for antenna applications with varying degrees of success in improving antenna characteristics. Some of these geometries have been particularly useful in reducing the size of the antenna, while other designs aim at incorporating multi-band characteristics. These are low profile antennas with moderate gain and can be made operative at multiple frequency bands and hence are multi-functional.

1.2 BACKGROUND

Fractal antennas are extension of classical antennas which employ fractal geometry. Thus, the fractal geometry and the antenna theory form the background of fractal antennas. These are discussed briefly as under

1.2.1 Fractal geometry

Fractal geometry was first discovered by Benoit Mandelbrot as a way to mathematically define structures whose dimension can not be limited to whole numbers. It is that branch of mathematics which studies properties and behavior of fractals [1]. These geometries have been used to characterize objects in nature that are difficult to define with Euclidean geometries including length of coastlines, branches of trees etc. These geometries have been used to characterize structures in nature that were difficult to define with Euclidean geometries.

1.2.1.1 Measurement of fractals

The usual way of measuring a fractal is usually done by some form of dimension which is a fraction or non integer. Dimension form an important part of the fractal measurement because most of the fractal aspects of an object are reflected by its dimension.

1.2.1.2 Definition of Fractal

According to Webster's Dictionary a fractal is defined as being "derived from the Latin fractus meaning broken, uneven: any of various extremely irregular curves or shape that repeat themselves at any scale on which they are examined."

1.2.1.3 Dimension of Fractal

Another definition of Fractals is “A fractal is by definition a set for which the Hausdorff-Besicovitch dimension strictly exceeds the topological dimension.
Dimension of geometry can be defined in several ways. Some examples are topological dimension, Euclidean dimension, self-similarity dimension and Hausdorff dimension. Some of these are special forms of Mandelbrot’s definition of the fractal dimension like self-similar dimension. If there are $n$ such copies of the original geometry scaled down by a fraction $f$, the similarity dimension $D$ is defined as:

$$D = \frac{\log n}{\log\left(\frac{1}{f}\right)} \quad (1.1)$$

1.2.1.4 Specification

This involves an efficient way of defining a fractal, for example an Iterated Function Systems (IFS) can be used to specify a fractal of certain classes.

1.2.1.5 Properties of fractal

Fractals represent a class of geometry with very unique properties including:

(i) Self-similarity

(ii) Fractional dimension

(iii) Formation by iteration

(iv) Space-filling

These properties can further be exploited to design antennas which are miniaturized, have improved input matching ability and are multi band/wideband.

1.3 Engineering Applications of Fractals

Ever since they were mathematically re-invented by Mandelbrot, fractals have found widespread applications in several branches of science and engineering. Disciplines such as geology, atmospheric sciences, forest sciences, physiology have benefited significantly by fractal modeling of naturally occurring phenomena [7]. Several books and monographs are available on the use of fractals in physical sciences. Fracture mechanics is one of the areas of engineering that has benefited significantly from the application of fractals. The space filling nature of fractal geometries has invited several innovative applications. Fractal mesh generation has
been shown to reduce memory requirements and CPU time for finite element analysis of vibration problems.

One area of application that has impacted modern technology most is image compression using fractal image coding. Fractal image rendering and image compression schemes have led to significant reduction in memory requirements and processing time. In electromagnetic, scattering and diffraction from fractal screens have been studied extensively. The self-similarity of the fractal geometry has been attributed to the dual band nature of their frequency response. Fractal antenna arrays and fractal shaped antenna elements have evolved in 1990’s

1.4 Antenna Engineering

The antenna (aerial, EM radiator) is a device, which radiates or receives electromagnetic waves. The antenna is the transition between a guiding device (transmission line, waveguide) and free space (or another usually unbounded medium). Its main purpose is to convert the energy of a guided wave into the energy of a free space wave (or vice versa) as efficiently as possible, while in the same time the radiated power has a certain desired pattern of distribution in space [2].

Many different structures can act as antennas. Generally, antennas are constructed out of conducting material of some nature and can be constructed in many shapes and sizes. The size is related to the wavelength of operation of the antenna. An antenna designed for operation at 10 kHz is almost always much larger than an antenna designed for operation at 10 GHz, for example. Transmission lines are used to guide the power from the transmitter to the antenna and should be impedance matched to both the transmitter and the antenna. The antenna forms a critical component in a wireless communication system. A good design of the antenna can relax system requirements and improve its overall performance. There are many different parameters that are used to characterize antennas [2]. Some of these are:

1.4.1 Effective Height

- The effective height of an antenna represents the effectiveness of an antenna as radiator or collector of electromagnetic wave energy.
- It indicates how far an antenna is the effective in transmitting and receiving the electromagnetic energy.
1.4.2 Gain and Directivity

- Gain is used to describe an antenna's ability to make the apparent power greater than the actual transmitted power in a given direction.
- Directivity is used to characterize an antenna. Directivity is defined as the ratio of the maximum radiation intensity to the average radiation intensity.
- Gain is equal to directivity if the efficiency of the antenna is 100 percent.
- Gain is a directional function; it changes with position around the antenna and is defined as

\[ G(\theta, \phi) = 4\pi \frac{U(\theta, \phi)}{P_{in}} \]  

(1.2)

Where \( U(\theta, \phi) \) is the radiation intensity and \( P_{in} \) is the input power to the antenna.

- Gain is usually measured in decibels with reference to another antenna either an isotropic radiator or to a simple dipole.
- An isotropic radiator is an antenna that radiates equally in all directions and is just a theoretical model.

1.4.3 Front to-Back Ratio (FBR)

- Front to back isolation ratio is defined as the difference in gain from the front of the antenna and the gain from the back of the antenna.
- FBR is of concern to communication engineers when the antenna is to be used in a crowded frequency band. Amateur radio operators frequently use front-to-back isolation as a parameter when comparing Yagi-Uda antennas.

1.4.4 Input Impedance

- The input impedance of the antenna should be matched to the impedance of the transmission line for maximum power transfer because when the impedance is purely resistive, the antenna dissipates the power presented to it.
- It is also important that the input impedance of the antenna is mostly resistive, so that most of the power introduced to the antenna is radiated. Input impedance has real and complex parts and its general form is:
\[ Z_{in} = R_{in} + jX_{in} \]  \hspace{1cm} (1.3)

Where \( R_{in} \) represents the resistance or power radiating portion of the impedance, \( X_{in} \) represents the reactive portion or power storage component of the impedance.

- Power can be dissipated from an antenna in two of the following ways:
  - Ohmic or heating losses from the antenna structure.
  - Power that leaves the antenna as electromagnetic waves at the desired frequency is another form of dissipation.

In some antennas, the ohmic losses are very small compared to the radiation losses. Non-zero capacitive or inductive reactance present non-radiating, energy storing fields that reduce the total radiated power of the antenna.

1.4.5 Voltage Wave Standing Ratio (VSWR)

- The voltage standing wave ratio (VSWR) is a measure of impedance match or mismatch between the transmission line and antenna.
- A VSWR of 1:1 indicates a perfect match, while a VSWR of \( \infty : 1 \) indicates the worst case.

1.4.6 Frequency Bandwidth (FBW)

- The bandwidth of an antenna is important in determining the frequency range and the application it can be used for. For example, a commercial radio transmission antenna can have a very narrow bandwidth because it will probably be used on only one frequency. A receiver antenna, however, must have a fairly large bandwidth to allow it to operate across many different frequencies.
- Antennas form three classes in terms of frequency coverage:
  - **Narrowband** - These antennas cover a small range of the order of few percent around the designed operating frequency.
  \[
  FBW = \frac{f_{\text{max}} - f_{\text{min}}}{f_0} \times 100\% \]  \hspace{1cm} (1.4)
  
  Where, \( f_{\text{max}} \), \( f_{\text{min}} \) are the maximum and minimum frequencies
  
  \( f_0 \) is the centre frequency
- *Wideband or broadband* – these antennas cover an octave or two range of frequencies.

\[ FBW = \frac{f_{\text{max}}}{f_{\text{min}}} \]  

- *Frequency Independent* - These antennas cover a ten to one or greater range of frequencies.

1.4.7 Radiation Pattern

- The radiation pattern (RP) (or antenna pattern) is the representation of the radiation properties of the antenna as a function of space coordinates.
- RP is measured in the far-field region, where the spatial (angular) distribution of the radiated power does not depend on the distance.
- The radiation pattern plot is useful for quickly evaluating the usefulness of an antenna for a certain application.

These parameters form a language and an important tool used to describe and compare antennas against one another. These parameters also allow a system designer to choose an antenna that is most suitable for their situation. For example, Gain, directivity are parameters that a radio systems engineer would use to choose an antenna for a specific job, i.e. an omni-directional antenna would be used for wide area coverage, like for a television transmitter, while an antenna with a narrow beam width would probably be used as a television receiver antenna because of its large gain in one direction and its ability to screen out interference from the sides and back.

1.5 Fractals in Antenna Engineering

The primary motivation of fractal antenna engineering is to extend antenna design and synthesis concepts beyond Euclidean geometry. In this context, the use of fractals in antenna array synthesis and fractal shaped antenna elements have been studied. Obtaining special antenna characteristics by using a fractal distribution of elements is the main objective of the study on fractal antenna arrays. It is widely known that properties of antenna arrays are determined by their distribution rather than the properties of individual elements. Since the array spacing (distance between
elements) depends on the frequency of operation, most of the conventional antenna array designs are band-limited. Self-similar arrays have frequency independent multiband characteristics. Fractal and random fractal arrays have been found to have several novel features. Variation in fractal dimension of the array distribution has been found to have effects on radiation characteristics of such antenna arrays. The use of random fractals reduces the fractal dimension, which leads to a better control of side lobes. Synthesizing fractal radiation patterns has also been explored. It has been found that the current distribution on the array affects the fractal dimension of the radiation pattern. It may be concluded that fractal properties such as self-similarity and dimension play a key role in the design of such arrays.

1.6 Fractal Shaped Antenna Elements

As with several other fields, the nature of fractal geometries has caught the attention of antenna designers, primarily as a past-time. However with the deepening of understanding of antennas using them several geometrical and antenna features have been inter-linked. This has led to the evolution of a new class of antennas, called fractal shaped antennas. Cohen, who later established the company Fractal Antennas. He has tried the usefulness several fractal geometries experimentally. Koch curves, Murkowski curves, Sierpinski gasket are among them. These geometries have a large number of tips and corners, a fact that would help improve antenna efficiency. Fractal trees were explored for the same reason, and were found to have multiband characteristics. Self-similarity of the geometry is qualitatively associated with the multiband characteristics of these antennas. Fractal shaped dipole antennas with Koch curves are generally fed at the center of the geometry. By increasing the fractal iteration, the length of the curve increases, reducing the resonant frequency of the antenna. The resonance of monopole antennas using these geometries below the small antenna limit has been reported by Puente et al. They have also studied the shift in resonant frequency by increasing the fractal iteration order. However detailed studies indicated that this reduction in resonant frequency does not follow the same pace as the increase in length with each subsequent iteration. As the fractal iteration is
increased the feature length gets smaller. There seems to be a limit in the minimum feature length that influences antenna properties.

Several other self-similar geometries have also been explored for multiband antenna characteristics. Sierpinski gaskets have been studied extensively for monopole and dipole antenna configurations. The self-similar current distribution on these antennas is expected to cause its multi-band characteristics. It has been found that by perturbing the geometry (thereby removing its self-similarity) the multiband nature of these antennas can be controlled. Variation in the flare angle of these geometries has also been explored to change the band characteristics of the antenna. Efforts have also been made to improve bandwidths of these antennas. A stacked antenna configuration with multiple layers of fractal geometries has been found to have some effect in this regard. This configuration has also been made conformal to improve the utilization of the antenna. Similar to Sierpinski gaskets, Sierpinski carpets have also been used in antenna elements. This geometry is also used as microstrip patch antenna with multiband characteristics. The antenna characteristics such as the peak gain are reported to improve by replacing a rectangular geometry with this fractal. To summarize the survey on fractal antenna engineering, key aspects of using fractals in antennas are presented here. For fractal arrays, several novel synthesizing algorithms have been developed to tailor radiation patterns. It has been established that random fractals can be used for better control of side lobe levels. Multi-band operation and a certain extent of frequency independence are possible with such array designs. The advantages of using fractal shaped antenna elements are manifold. These geometries can be used to design smaller sized resonant antennas. The antenna radiation efficiency’s thought to have improved by large number of bends and corners in many of such fractals. These geometries can lead to antennas with multi-band characteristics, often with similar radiation characteristics in these bands.

1.7 Features of Fractal Antennas

As already mentioned the fractal antennas employ the fractal geometry for their design as compare to classical antennas which employ Euclidean Geometry. The two basic properties of fractals provide distinguish features to these fractal designed antennas, these are discussed with appropriate application areas below:
1.7.1 Multiband/ Wideband performance

Any good antenna system requires antenna scaling which means that the different parameters (impedance, gain, pattern etc.) remain same if all the dimensions and the wavelength are scaled by same factor. Since due to self-similarity possessed by fractals, the fractal structure appear to be same independent of size scaling and thus it can be interpreted that the fractal structures can be used to realize antenna designs over a large band of frequencies [8]. The antenna can be operated similarly at various frequencies which mean that the antenna keeps the similar radiation parameters through several bands.

Application: In modern wireless communications more and more systems are introduced which integrate many technologies and are often required to operate at multiple frequency bands. Thus demands antenna systems which can accommodate this integration. Examples of systems using a multi-band antenna are varieties of common wireless networking cards used in laptop computers. These can communicate on 802.11b networks at 2.4 GHz and 802.11g networks at 5 GHz. Use of fractal self-similar patterns offers solution.

1.7.2 Compact Size

Another requirement by the compact wireless systems for antenna design is the compact size. The fractional dimension and space filling property of fractal shapes allow the fractal shaped antennas to utilize the small surrounding space effectively [9]. This also overcomes the limitation of performance of small classical antennas.

Application: The fractal antenna technology can be applied to cellular handsets. Because fractal antennas are more compact, they fit more easily in the receiver package. Currently, many cellular handsets use quarter wavelength monopoles which are essentially sections of radiating wires cut to a determined length. Although simple, they have excellent radiation properties. However, for systems operating at high frequencies such as GSM, the length of these monopoles is often longer than the handset itself. It would be highly beneficial to design an antenna based on fractal design with similar radiation properties as the quarter wavelength monopole while
retaining its radiation properties. This designed antenna will fit in a more compact manner.

1.8 Advantages and Disadvantages

The various advantages of fractal antennas can be listed as:

- Smaller cross sectional area
- No impedance matching network required
- Multiple resonances
- Higher gain in some cases

Though in the early stage of their development these antenna designs suffer from two main disadvantages. These are:

- Fabrication and design is little complicated
- Lower gain in some cases

Further investigations and new developments in this field may be helpful in overcoming these disadvantages.

1.9 Objective of the thesis

The current thesis work is carried out to meet the following objectives:

- To generate Koch fractal, Sierpinski Triangle, Sierpinski Carpet fractal for different iterations using MATLAB.
- Antenna miniaturization using 2.1 cm Koch fractal of Second iteration for GSM 900, GSM 1800.
- Comparison of 2.1 cm Koch fractal of second iteration with its array for GSM 900, GSM1800.

1.10 Methodology

PC Configuration: Intel Pentium 4,
1.60 GHz,
256 MB of RAM
Operating System: Windows XP Professional, Version 2002 (Service pack 2)
Software: MATLAB 6.1
MMANA (Antenna Analyzing Tool)
1.11 Outline of Thesis

The thesis is organized in six chapters.

Chapter 1 provides an introduction to fractal antennas, their emergence and basic features. The fundamental concepts of fractal, their properties, generation and common fractals structures are presented in Chapter 2.

The use of fractals and fractal geometry in design of antenna systems and their approach as antennas is discussed in Chapter 3. The various limitations of classical antennas are also presented. A brief discussion on different fractal antennas and their development in recent years is also summarized. Designing of the Koch fractal monopole is described in Chapter 4.

In Chapter 5 Design and simulation of three elements Koch linear antenna array is presented and various parameters like input impedance, gain, front back ratio, and bandwidth are simulated at GSM 900 and results are compared with single element Koch antenna. Same parameters of Koch linear antenna array are simulated at GSM 1800 and results are compared with single element Koch antenna. Chapter 6 concludes the thesis work and discusses future scope of work.
Chapter 2
Literature Review

2.1 Introduction

This chapter presents a literature review of the theory of fractal antennas. The word fractal, derived from the Latin word ‘fractus’ meaning ‘broken’, i.e., fragmented, fractional or irregular, was originally coined by Mandelbrot in the early 1970's [5]. A fractal is a rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole. Fractals are generally self-similar and independent of scale. Thus, a fractal is a geometric shapes which:

(i) is self-similar and

(ii) has got fractional dimension

These geometries have been used to characterize structures in nature that were difficult to define with Euclidean geometries. Examples include the length of a coastline, the density of clouds, and the branching of trees. Just as nature is not confined to Euclidean geometries.

2.2 Fractal Theory

Fractals are a class of shapes which have no characteristic size. Each fractal is composed of multiple iterations of a single elementary shape. The iterations can continue infinitely, thus forming a shape within a finite boundary but of infinite length or area. This compactness property is highly desirable in mobile wireless communication applications because smaller receivers could be produced.

2.3 Some Useful Fractal Geometries

The term fractal was coined by the French mathematician B.B. Mandelbrot during 1970’s after his pioneering research on several naturally occurring irregular
and Fragment geometries not contained within the realms of conventional Euclidian geometry. The term has its roots in the Latin word *fractus* which is related to the verb *fang ere* (meaning: to break).

These geometries were generally discarded as formless, but Mandelbrot discovered that certain special features can be associated with them. Many of these curves were recognized well before him, and were often associated with mathematicians of yesteryears. But Mandelbrot’s research was path-breaking: he discovered a common element in many of these seemingly irregular geometries and formulated theories based on his findings.

Some of the more common fractal geometries that have been found to be useful in developing new and innovative designs for antennas. The first fractal that will be considered is the popular sierpinski gasket shown in Fig 2.1 the procedure for geometrical constructing this fractal begins with an equilateral triangle contained in the plane as illustrated in stage 0 of Fig 2.1. The next step in the construction process (see stage 1 of Fig 2.1) is to remove the central triangle with vertices that are located at the midpoints of the sides of the original triangle, shown in stage 0. This process is then repeated for the three remaining triangles, as in stage 2 of Fig 2.1. The next two stages (stages 3 and 4) in the construction of the sierpinski gasket are also shown in Fig 2.1. The sierpinski gasket fractal is generated by carrying out this iterative process an infinite number of times. It is easy to see from this definition that the sierpinski gasket is an example of a self similar fractal. Fig 2.1 black triangular areas represent a metallic conductor, whereas the white triangular areas represent regions where metal has been removed.

Another popular fractal is known as the Koch snowflake. This fractal also starts out as a solid equilateral triangle in the plane, as illustrated in stage 0 of Fig 2.2
However unlike the sierpinski gasket, which was formed by systematically removing smaller and smaller triangles from the original structure, the Koch snowflake is constructed by adding smaller and smaller triangles to the structure in an iterative fashion. This process is clearly represented in Fig 2.2 where the first few stages in the geometrical construction of a Koch snowflake are shown.
Figure 2.2- The First Few Stages in the Construction of a Koch Snowflake

A number of structure based on purely deterministic or random fractal trees have also proven to be extremely useful in developing new design methodologies for antennas. Fig 2.3 this particular ternary tree structure is closely related to the sierpinski gasket shown in Fig 2.1

Figure 2.3- A Stage 4 Ternary Fractal Trees
The space filling properties of the Hilbert curve and related curves make them attractive for the use in the design of fractal antenna. The first four steps in the construction of the Hilbert curve are shown in Fig 2.4

![Stage 0 and Stage 1 of the Hilbert curve](image)

**Figure 2.4- The First Few Stages in the Construction of a Hilbert Curve**

Some of the more common fractal geometries that have found application in antenna engineering are depicted in Fig 2.5 the Koch snowflakes and islands have been primarily used to develop new designs for miniatures lop as well as microstrip patch antennas. New designs for miniaturized dipole antennas have also been develop based on a variety of Koch curves and fractal trees. Finally the self similar structure of sierpinski gaskets and carpets has been exploited to develop multi band antenna elements.
2.4 Fractal’s Definition

Mandelbrot defines the term fractal in several ways. These rely primarily on the definition of their dimension. A fractal is a set for which the Hausdorff Besicovich
dimension strictly exceeds its topological dimension. Every set having non-integer
dimension is a fractal. But fractals can have integer dimension. Alternately, fractal is
defined as set F such that.

- F has a fine structure with details on arbitrarily small scales.
- F is too irregular to be described by traditional geometry.
- F having some form of self-similarity (not necessarily geometric, can be
  Statistical).
- F can be described in a simple way, recursively, and Fractal dimension of F
greater than its topological dimension.

Topological dimension, Euclidean dimension, self-similarity dimension and
Hausdorff dimension are the dimension of Fractal geometry. Some of these are
special forms of Mandelbrot’s definition of the fractal dimension. However the most
easily understood definition is for self-similarity dimension. To obtain this value, the
geometry is divided into scaled down, but identical copies of itself. If there are n such
copies of the original geometry scaled down by a fraction $f$, the similarity dimension
$D$ is defined in eq (1.1).

For example, a square can be divided into 4 copies of $\frac{1}{2}$ scale, 9 copies of $\frac{1}{3}$
scale, 16 copies of $\frac{1}{4}$ scale, or $n^2$ copies of $\frac{1}{n}$ scale. Substituting in the above formula,
the dimension of the geometry is ascertained to be 2. The same approach can be
followed for determining the dimension of several fractal geometries.

Although this approach is very convenient for much such geometry, all
fractals are not amenable for this approach. Such is the case with most plane-,
or space-filling fractals. In these cases more mathematically intensive definitions such as
Hausdorff dimension are required. Non-integer dimension is not the only peculiar
property of fractals. A glossary of terms used in describing properties of fractals is
introduced next. A self-similar set is one that consists of scaled down copies of itself.
Many fractal geometries are self-similar, a property which makes easier to accurately
compute their Hausdorff dimension. In order to define self-similarity mathematically,
first the concept of contraction is introduced. A map $\psi: \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a contraction if
there exists some constant number $c \in (0, 1)$ so that the inequality

$$|| \varphi(x) - \varphi(y) || \leq c \ || x - y ||$$

(2.2)
hold for any \( x, y \in \mathbb{R}^n \). for a natural number \( m \geq 2 \), and a set of \( m \) contractions \( \{ \varphi_1, \varphi_2, \ldots, \varphi_m \} \) defined on \( \mathbb{R}^n \), a non empty compact set \( v \) in \( \mathbb{R}^n \) is self similar if

\[
V = \bigcup_{i=1}^{m} \varphi_i (v)
\]  

(2.3)

called Self-affine sets. Other properties associated with fractal geometries include scaleinvariance, plane-filling or space-filling nature, and lacunarity. Lacunarity is a term coined to express the nature of area fractal having hollow spaces. Plane filling fractals are those that tend to fill an area (or space, in more general terms) as the order of iteration is increased. Some of these properties are qualitatively linked to the features of antenna geometries using them. It is envisaged that the above description of these properties would shed light into a better understanding of such connection. In the following sub-sections a brief introduction is provided on the use of fractal in science and engineering, and antenna engineering in particular.

### 2.5 Iterated Function Systems

Many useful fractals can be generated by Iterated Function Systems (IFS) [3]. An extended discussion of IFS is found in. Briefly, IFS work by applying a series of affine transformations \( W \) to an elementary shape an over much iteration. The affine transformation \( W(x, y) \), comprising rotation, scaling and translation, is given by:

![Figure 2.6- A 4-Iterations Sierpinski Gasket](image)
Affine transformation $W(x,y)$ is represented by six parameters $a, b, c, d, e, f$ where $a, b, c, d$ are rotation and scaling parameter and $e, f$ are translation parameter.

$$W(x, y) = \begin{bmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ (2.4)

The set of affine transforms $W(A)$, known as the Hutchinson operator is given by:

$$W(A) = \bigcup_{i=1}^{N} w_i(A) = w_1(A) \bigcup w_2(A) \bigcup w_3(A) \ldots \bigcup w_N(A)$$ (2.5)

The fractal then can be generated by applying operator $W$ to the previous geometry for $k$ iterations thus:

$$A_1 = W(A_0), A_2 = W(A_1) \ldots A_{k+1} = W(A_k)$$ (2.6)

### 2.5.1 The Sierpinski Gasket Fractal

The Sierpinski Gasket fractal is generated by the IFS method. As depicted in Figure 2.6 a triangular elementary shape is iteratively scaled, rotated and translated, then removed from the original shape in order to generate a fractal. It is interesting to note that after infinite iterations of the fractal, the entire shape has an infinite area but is bounded by a finite perimeter.

#### 2.5.1 (a) Mod –p sierpinski gaskets

Sierpinski gasket is a special case of a wider class of fractals that can be derived from the well known Pascal’ triangle [6]. This class of fractals can be derived in the following way. Consider an equiangular triangular grid whose rows shall be labeled by $n = 1, 2, 3 \ldots$ each row contain $n$ nodes and to each node a number is attached. This number is the coefficient of the binomial expansion of $(x+y)^{n-1}$. Now delete from this grid those nodes that are attached to numbers that are exactly divisible by $p$, where $p$ is a prime number. The result is a self similar fractal that will be referred as the mod-p sierpinski gasket in Fig2.7, this process is shown for the
mod-2 sierpinski gasket and the blank nodes represent those deleted from the grid. To obtain the mod-3 and mod-5 sierpinski gaskets, those nodes attached to numbers divisible by 3 and 5 should be deleted from the Pascal’s. In Fig 2.7b three iteration mod-3 and mod -5 sierpinski gaskets are shown. In this case it is clear that the scaling of the different replicas is p. this antenna is basically used for fractal multi band antenna.

![Figure 2.7(a) - Derivation of the Sierpinski Gasket from Pascal’s triangle. When those numbers divisible by 2 are deleted the mod-2 sierpinski gasket is obtained.](image)

2.5.2 The Koch Fractal

The Koch fractal curve is one of the most well-known fractal shapes. It consists of repeated application of the series of IFS affine transformations given in (2.4). Multiple iterations of the Koch fractal are shown in Figure 2.2. To form the first iteration \( n = 1 \) in Figure 2.2, the affine transform \( w_1 \) scales a straight line to one third of its original length. The transform \( w_2 \) scales to one-third and rotates by 60 degree. The third transform, \( w_3 \) is similar to \( w_2 \) but rotating by -60. Finally the fourth
transform, \( w_4 \) is simply another scaling to one-third and a translation. It can be seen in Figure 2.2 how these sets of transforms are applied to each previous iteration to obtain the next.

\[
\begin{align*}
\begin{bmatrix}
\frac{1}{3} & 0 & 0 \\
0 & \frac{1}{3} & 0 \\
0 & 0 & 1
\end{bmatrix} & \quad w_1 = \\
\begin{bmatrix}
\frac{1}{3} \cos 60 & \frac{1}{3} \sin 60 & \frac{1}{3} \\
\frac{1}{3} \sin 60 & \frac{1}{3} \cos 60 & 0 \\
0 & 0 & 1
\end{bmatrix} & \quad w_2 = \\
\begin{bmatrix}
\frac{1}{3} \cos 60 & \frac{1}{3} \sin 60 & \frac{1}{3} \\
\frac{1}{3} \sin 60 & \frac{1}{3} \cos 60 & 0 \\
0 & 0 & 1
\end{bmatrix} & \quad w_3 = \\
\begin{bmatrix}
\frac{1}{3} & 0 & 0 \\
\frac{1}{3} & \frac{2}{3} & 0 \\
0 & \frac{1}{3} & 1
\end{bmatrix} & \quad w_4 = \\
\end{align*}
\]

An important characteristic of the Koch fractal worthy of note is that the unfolded length of the fractal approaches infinity as the number of iterations approach infinity. However, the area which bounds the fractal remains constant [3]. This property can be used to minimize the space use of a simple wire monopole or dipole antenna.

### 2.6 Why Fractals are space filling geometries

Euclidean geometries are limited to points, lines, sheets & volumes, Fractal include geometries that fall in between these distinctions. Therefore, a fractal can be line that approaches a sheet. These space filling properties lead to curve that are electrically very long [19], but fit into a compact physical space. This property leads to miniaturization of antenna elements. Fractals could be used to define the spacing in arrays for thinning or to define radiation pattern [20]. With successive iteration the length of Koch increases by 1/3 of the original length. Length of Koch after nth iterations:
\[ l_n = l_0 \left( \frac{4}{3} \right)^n \]  
(2.8)

Where \( l_n \) and \( l_0 \) are the length after nth iteration and original length (without any iteration) respectively. For Sierpenski Triangle with each iteration the area of the holes and circumference of solid pieces changes. If the area of original triangle is 1, then first iteration removes 1/4 of the area. Second iteration removes a further 3/16 and third iteration 9/64. Then total area removed after the Nth iteration

\[ A_N = 1/3 \sum_{i=1}^{N} (3/4)^i \]  
(2.9)

\[ A_\infty = 1 \]  
(2.10)

If circumference of original triangle is 1, then after first iteration the circumference increases by 1/2. After second iteration it increases by 3/4, after nth iteration

\[ C_N = 1 + 1/3 \sum_{i=1}^{N} (3/2)^i \]  
(2.11)

And 

\[ C_\infty = \infty \]  
(2.12)

This means gasket has no area but boundary is of infinite length. Figure 2.8 shows how with each iteration the area of holes and circumference.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Area</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Triangle" /></td>
<td>( A_0 = \sqrt{3}/4 )</td>
<td>( P_0 = 3 )</td>
</tr>
</tbody>
</table>
| ![Triangle](image) | \( A_1 = 3/4 A_0 \) | \( P_1 = 3 + 3(1/2) \)  
\[ = 3 + 3/2 \] |
| ![Triangle](image) | \( A_2 = (3/4)^2 A_0 \) | \( P_2 = 3 + 3/2 + 3*3*1/4 \)  
\[ = 3 + 3/2 + 9/4 \] |
A₀ &=& (3/4)^3 A₀ \nonumber \\
A₀ &=& (3/4) \nonumber \\
P_3 &=& 3 + 3/2 + 9/4 + 9*3*1/8 \\
&=& 3 + 3/2 + 9/4 + 27/8 \\

<table>
<thead>
<tr>
<th>Stage n</th>
<th>A_n = (3/4)^n A₀</th>
<th>P_n = 3 + 3/2 + ... + (3/2)^n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sierpenski Triangle</td>
<td>0</td>
<td>infinity (geometric series with r &gt; 1)</td>
</tr>
</tbody>
</table>

Figure 2.8 - Different iteration of Gasket and variation of area and circumference

[18]

2.7 Fractals in nature and Applications

Fractals are not just complex shapes and pretty pictures generated by computers. Anything that appears random and irregular can be a fractal. Fractals permeate our lives, appearing in places as tiny as the membrane of a cell and as majestic as the solar system. Fractals are the unique, irregular patterns left behind by the unpredictable movements of the chaotic world at work. In theory, one can argue that everything existent on this world is a fractal [27].

Fractals have more and more applications in science.

**Astronomy**

Fractals will maybe revolutionize the way that the universe is seen. Cosmologists usually assume that matter is spread uniformly across space. But observation shows that this is not true. Astronomers agree with that assumption on "small" scales, but most of them think that the universe is smooth at very large scales. However, a dissident group of scientist’s claims that the structure of the universe is fractal at all scales.
Nature

Take a tree, for example. Pick a particular branch and study it closely. Choose a bundle of leaves on that branch. All three of the objects described - the tree, the branch, and the leaves - are identical. To many, the word chaos suggests randomness, unpredictability and perhaps even messiness. Weather is a favorite example for many people. Forecasts are never totally accurate, and long-term forecasts, even for one week, can be totally wrong. This is due to minor disturbances in airflow, solar heating, etc. Each disturbance may be minor, but the change it creates will increase geometrically with time. Soon, the weather will be far different than what was expected. With fractal geometry we can visually model much of what we witness in nature, the most recognized being coastlines and mountains. Fractals are used to model soil erosion and to analyze seismic patterns as well.

Computer science

Actually, the most useful use of fractals in computer science is the fractal image compression. This kind of compression uses the fact that the real world is well described by fractal geometry. By this way, images are compressed much more than by usual ways (e.g.: JPEG or GIF file formats). An other advantage of fractal compression is that when the picture is enlarged, there is no pixelisation. The picture seems very often better when its size is increased.

Fluid mechanics

The study of turbulence in flows is very adapted to fractals. Turbulent flows are chaotic and very difficult to model correctly. A fractal representation of them helps engineers and physicists to better understand complex flows. Flames can also be simulated. Porous media have a very complex geometry and are well represented by fractal. This is actually used in petroleum science.

Surface physics

Fractals used to describe the roughness of surfaces. A rough surface characterized by a combination of two different fractals.

Medicine

Biosensor interactions can be studied by using fractals.
Telecommunications

A new application is fractal-shaped antenna that reduces greatly the size and the weight of the antennas. The benefits depend on the fractal applied, frequency of interest, and so on. In general the fractal part produces ‘fractal loading’ and makes the antenna smaller for a given frequency of use. Practical shrinkage of 2-4 times are realizable for acceptable performance. Surprisingly high performance is attained.
CHAPTER 3
FRACTAL ANTENNA ELEMENT

3.1 Introduction

A wide variety of applications of fractals can be found in many branches of science and engineering. One such area is fractal electrodynamics. Fractal geometry can be combined with the electromagnetic theory for the purpose of investigating a new class of radiation, propagation and scattering problems.

One of the most promising areas of fractal electrodynamics research is in its application to antenna theory and design [8]. There are a variety of approaches that have been developed over the years, which can be utilized to achieve one or more of these design objectives pertaining to size, gain, efficiency and bandwidth. Unique properties of fractals can be exploited to develop a new class of antenna element designs that are multi-band, compact in size and can possess several highly desirable properties, including multi-band performance, low side lobe levels, and their ability to develop rapid beam forming algorithms based on the recursive nature of fractals.

Recent progress in the study of fractal antennas suggests some attractive solutions for two of the main limitations of the classical antennas, which are the single band performance and the dependence between size and operating frequency [22].

Fractals make possible the use of a single small antenna operating in several frequency bands. The self-similar properties of certain fractals result in a multiband behavior of the antennas while, the highly convoluted shape of these fractals makes possible the reduction in size, and consequently in mass and volume of certain antennas [4]. Fractal shapes radiate signals at multiple frequency bands, occupy space more efficiently and offer design solutions meeting the requirements for antennas in future wireless devices. These reductions can make possible to combine multimedia, communication and teledetection functionalities in a reduced space like a handy phone, a wristwatch or a credit card e.g. a fractal antenna can provide GPS (Global Positioning System) services within a conventional mobile cellular phone.
3.2 Limitations on Small Antennas

With fast growing development of wireless communication systems there has been an increasing need for more compact and portable communications systems. Just as the size of circuitry has evolved to transceivers on a single chip, there is also a need to evolve small sized, high-performance and low cost antenna designs which are capable of adjusting frequency of operation for integration of multiple wireless technologies and decrease in overall size. However when the size of the classical antenna (designed using Euclidean geometry) is made much smaller than the operating wavelength it becomes highly inefficient because radiation efficiency and impedance bandwidth decrease with the size of the antennas because these effects are accompanied by high currents in the conductors, high ohmic losses and large values of energy stored in the antenna near field. Limits of an electrically small antenna can be analyzed by assuming the antenna to be enclosed with a radian sphere of radius \( a \) [13]. The limit for the smallest possible quality factor, \( Q \) for any antenna within the radian sphere regardless of its shape can be described as:

\[
Q = \frac{1 + 2(ka)^2}{(ka)^3 \left(1 + (ka)^2\right)} \tag{3.1}
\]

An antenna is said to be small when it can be enclosed into a radian sphere, i.e. a sphere with radius \( a \), where \( a = \lambda / 2\pi \). Due to the variations of the current inside, the radian sphere the field outside the radian sphere can be described as a set of orthogonal spherical vector waves. For such antennas a fundamental limitation on the \( Q \) is established by Chu as:

\[
Q = \frac{1}{k^3 a^3} + \frac{1}{ka} \tag{3.2}
\]

This forms the lower fundamental limit of the \( Q \) factor that can be achieved by a linearly polarized antenna and is established regardless of the antenna current distribution inside the sphere. The current distribution inside the sphere is not uniquely determined by the field distribution outside the sphere so several current distributions can lead to the same \( Q \) factor. Here \( Q \) is described according to the stored electric energy \( W_e \), magnetic energy \( W_m \), frequency \( w \) and average radiated power \( P_r \) as:
\[ Q = \frac{2W_c}{P_r}, \quad W_c >> W_m \]  
\[ Q = \frac{2W_m}{P_r}, \quad W_c >> W_m \]  \hspace{1cm} (3.3)

An infinitesimally small antenna radiates only a popular fractal is known as the Koch snowflake. This fractal also starts out as a solid equilateral triangle in the plane, as illustrated in stage 0 of Fig 2.2 or \( TM_{01} \) spherical mode that depends on the electric size of the antenna given by \( ka \), where \( k \) is the wave number at resonance and \( a \) is the radius of the smallest sphere that encloses the antenna [2]. The real power is radiated because of the propagating modes, while the reactive power is due to all modes. However when this radian sphere becomes very small there are no propagating modes and only less real power. Further the radiation resistance decreases while proportionally the reactive energy stored in the antenna neighborhood increases rapidly which contributes to larger \( Q \) values. In general the \( Q \) of an antenna is inversely proportional to its bandwidth thus implying narrow bandwidth for the antennas with high values of \( Q \). Narrow bandwidth antennas are not usually preferred because of the difficulty of matching. Achieving a low \( Q \) antenna basically depends on how efficiently it uses the available volume inside the radian sphere. Thus the high currents in the conductors, high ohmic losses, large values of the stored energy in the antenna near field and high \( Q \) values make the performance of small antennas inefficient.

3.3 Fractals as Antenna Elements

Small antennas are of prime importance because of the available space limitation on devices and the oncoming deployment of diversity and multi-input multi-output (MIMO) systems. The basic antenna miniaturization techniques can be summarized into lumped-element loading, material loading, and use of ground planes, short circuits, the antenna environment and finally the antenna geometry. Among these techniques the antenna geometry optimization and use of ground planes can achieve miniaturization or compactness of the antenna while maintaining the good
antenna performance in terms of bandwidth and efficiency. However the classical small antennas suffer from inefficient performance. Fractal geometry provides the solution by designing compact and multiband antennas in a most efficient and sophisticated way. The general concepts of fractals can be applied to develop various antenna elements [5]. The properties of these fractal designed antennas allows for smaller, resonant antennas that are multiband and may be optimized for gain. When antenna elements or arrays are designed with the concept of self-similarity for most fractals, they can achieve multiple frequency bands because different parts of the antenna are similar to each other at different scales. Application of the fractional dimension of fractal structure leads to the gain optimization of wire antenna and the self-similarity makes it possible to design antennas with very wideband performance.

3.3.1. Fractals as Miniaturized Antennas

Wire antennas miniaturization is usually based in packing a long wire inside a small volume with the aim to achieve the smallest antenna having a given resonant frequency or, equivalently, achieving the lowest resonant frequency of an antenna having a fixed size. In the miniaturization of wire antennas it has been found that the electromagnetic coupling between wire angles limits the reduction of the resonant frequency with increasing wire length. In principle, it is expected that the longer the wire length, the lower is the resonant frequency. Fractal geometry can be employed to design self resonant small antennas in which effective reduction in the resonant frequency can be obtained. It should be noted though applying fractal geometry to reduce the size of the wire antenna a reduction in resonant frequency is obtained. The effect can be explained with the help of Koch fractal curve to understand the behavior of the resonant frequency of fractal antennas as a function of the antenna geometry and wire length. It has been found that with increase in number of iterations, $n$, the effective length increases by a factor of $(4/3)^n$. Thus with an increase of the wire length of a Koch fractal there is a decrease in the resonant frequency. The observed behavior can be further explained due to the coupling fact between the sharp angles at curve segment junctions as shown in Figures 3.1 and 3.2. These angles radiate a spherical wave with phase center at the vertex. Each angle not only radiates, but also receives the signal radiated by other angles. As a consequence, part of the signal does not follow the wire
path, but takes shortcuts that start at a radiating angle. The length of the path traveled by the signal is, therefore, shorter than the total wire length. The degree of coupling between parallel wire segments with opposite current vectors causes a significant reduction in the effective length of the total wire, and therefore an increase in the resonant frequency. When used as wire antenna the fractal antennas leads to more effective coupling of energy from feeding transmission lines to free space in less volume. Similarly when used as loop antennas, the fractal antennas with increased length it raises the input resistance of a loop antenna.

3.3.2 Fractals as Multiband Antennas

It has been found that for an antenna, to work well for all frequencies i.e. show a wideband/multiband behavior, it should be:

- Symmetrical: This means that the figure looks the same as its mirror image.
- Self-similar: This means that parts of the figure are small copies of the whole figure.

These two properties are very common for fractals and thus make fractals ideal candidates for design of wideband /multiband antennas [22].

Traditionally a wideband/multiband antenna in the low frequency wireless band can only be achieved with heavily loaded wire antennas which usually imply that

Figure 3.1- Different Iterations of the Koch monopole [20]
different antennas are needed for different frequency bands [16]. Recent progresses in the fractal antennas suggest solution for using a single small antenna operating in several frequency bands. The self similarity properties of the fractal structures are translated into the electromagnetic behavior when used as antenna. This multiband behavior can be explained with the help of a Sierpinski triangle (gasket) antenna employing Sierpinski fractal geometry with a self similar structure. Figure 3.3 show a typical antenna system in which a single antenna is used for each application that is intended for each different frequency band (four bands in this figure). However, use of Sierpinski triangle (Gasket) allows a structure in which only single antenna is intended to be used for all of the four frequency bands as illustrated in Figure 3.4.

![Figure 3.2](image1.png)

**Figure 3.2- Four different antennas to be used for four different frequency bands [17]**

![Figure 3.3](image2.png)

**Figure 3.3- Single antenna used for four different frequency bands using the fractal geometry of Sierpinski triangle [17]**
3.3.3 Cost Effectiveness of Fractal Antennas

One practical benefit of fractal antenna is that it is resonant antenna in a small space thereby excluding the need of attaching discrete components to achieve resonance. Usually at UHF and microwave antenna the cost for such parts for the transceivers can become more expensive than the antenna [6]. Further the addition of parts produces reliability issues and breakage/return problems. In most of applications fractal antennas are small bendable etched circuit boards or fractal etchings on mother boards and contain no discrete components. This makes design of fractal antennas a cost effective technique [14].

3.4 Different Fractal Antennas

As discussed earlier the general concepts of fractals can be applied to develop various antenna elements. Several frequency independent antennas can be generalized as fractal antennas. Small antennas have several limitations in tradeoffs of directivity, bandwidth and radiation efficiency. Recent work on fractal elements suggests that apparently complex structures like fractals perform fairly well when their sizes are electrically small (i.e. far less than a wavelength in size, but not in the limit of zero size). Fractal antennas can take on various shapes and forms depending on the different fractal geometries. Antennas using some of these geometries for various telecommunications applications are already available commercially. Cohen was the first to develop an antenna element using the concept of fractals [4]. He demonstrated that the concept of fractal could be used to significantly reduce the antenna size without degrading the performance. The first application of fractals to antenna design was thinned fractal linear and planar arrays as wideband arrays and multiband performance was obtained by arranging elements in a fractal pattern to reduce number of elements in an array. The multiband capability of fractals by studying the behavior of the Sierpinski monopole and dipole was demonstrated by Puente et al [14]. The Sierpinski monopole displayed a similar behavior at several bands for both the input return loss and radiation pattern. Multiband or ultra wideband antennas can also be obtained by using other structures of fractals [24]. In some designs, fractal structures are used to achieve a single very wideband response, such as in printed circuit fractal loop.
Fractal antennas have small form factors for cell, 900 MHz, and S-band/PCS applications. This makes them a logical choice for antennas placed in/on casing of transceivers, receivers and transmitters where the additional loading is easily met by a slight scaling of the fractal pattern, further in this case the chances of breaking off an antenna that does not stick out of the casing is minimized. Some of the different types of fractal antennas which have been found in use are:

3.4.1 Koch Monopole and Dipole

It is one of the applications of fractal geometry in the design of wire antenna elements. The geometry of a standard dipole or loop antenna can be fractalised by systematically bending the wire in a fractal way, so that the overall arc length remains the same, and the size is correspondingly reduced with the addition of each successive iteration [19]. The Koch curve has been used to construct a monopole and a dipole in order to reduce antenna size shown in Figure 3.1. The miniaturization of the antennas shows a greater degree of effectiveness for the first several iterations. From the properties of the Koch fractal monopole it was shown that the electrical performance of Koch fractal monopoles is superior to that of conventional straight wire monopoles, especially when operated in the small-antenna frequency regime. Figure 3.5 shows a Koch monopole antenna.

![Figure 3.4- A Koch monopole](image)

Figure 3.4- A Koch monopole [20]
3.4.2. Koch Loop and Minkowski Loop

Loop antennas can be well understood by using a variety of Euclidean geometries. Resonant loop antennas require a large amount of space and small loops have very low input resistance however a fractal Koch Island can be used as a loop antenna to overcome these drawbacks [23]. The two commonly used loop antennas structures: fractal Koch Island and Minkowski loop are shown in Figure 3.6.

![Koch Loop](image1.png) ![Minkowski Loop](image2.png)

Figure 3.5- Fractal loop antennas (a) Koch Loop (b) Minkowski Loop [23]

3.4.3 Sierpinski Monopole and Dipole

This is one of the most popular fractal structure used for multiband performance and can be constructed from a triangle as illustrated in Figure 2.7. The self-similarity properties of the fractal shape results in a multiband antenna. A multiband fractal monopole antenna, based on the Sierpinski gasket with a flare angle of $\alpha$ and a self-similarity scale factor of $\delta$, was first introduced by Puente et al [26].

A particular scheme for modifying the spacing between the bands of the Sierpinski monopole can be adopted. The positions of the multiple bands may be controlled by proper adjustment of the scale factor used to generate the Sierpinski antenna. Also, the variation in the flare angle of the antenna can not only translate a shift of the operating bands, but also changes the input impedance and radiation patterns [26]. Figure 3.7 shows a Sierpinski monopole antenna system design.
3.4.4 Fractal Patch antennas

The design methodology and technique for a multiband Sierpinski microstrip patch antenna to improve the multiband behavior of the radiation patterns by suppressing the effects of high order modes [24]. A patch antenna with a Koch fractal boundary exhibits localized modes at a certain frequency above the fundamental mode, which can lead to broadside directive patterns. The localized modes were also observed in a waveguide having Koch fractal boundaries. The Cantor slot patch is another example of multiband fractal structure which has been applied in multiband microstrip antennas.

3.4.5 Printed Circuit fractal antennas

Printed circuit antennas are desired in many instances due to the space constrains in the modern electronic devices. The printed circuit antennas are useful for the easiness of construction and for the reduced occupied space. The printed circuit fractal loop antenna is designed to achieve ultra wideband or multiple wideband performance and to reduce the antenna dimensions. The antenna has a constant phase center, can be manufactured using printed circuit techniques, and is readily conformable to an airframe or other structure [21].

3.4.6 Fractal Antenna Arrays
The concept of the fractal can be applied in design and analysis of arrays by either analyzing the array using fractal theory or by placing elements in fractal arrangement or considering the both [12]. Fractal arrangement of array elements can produce a thinned array and can achieve multiband performance. Properties of random fractals were used to develop a design methodology for quasi-random arrays. Random fractals were also used to generate array configurations that were somewhere between completely ordered (i.e., periodic) and completely disordered (i.e., random). The main advantage of this technique is that it yields sparse arrays that possess relatively low sidelobes (a feature associated with periodic arrays, but not random arrays), and which are also robust (a feature associated with random arrays, but not periodic arrays) [10].

A family of nonuniform arrays, known as Weierstrass arrays have the property that their element spacing and current distributions are self-scalable and can be generated in a recursive fashion. Synthesis techniques for fractal radiation patterns were also developed based on the self-scalability property.
Chapter 4

Design of the Koch Fractal monopole

4.1 Introduction

This chapter presents the design of the Koch fractal monopole [4]. The purpose of building this monopole is to produce a more space-efficient quarter-wave monopole design while maintaining the radiation properties of the traditional quarter-wave, straight-wire monopole.

4.2 Selection of operating frequency

The operating frequency of 900 MHz was chosen for the design of the Koch fractal monopole. This frequency band is used for cellular wireless telephony through the GSM system. Also, the space-filling properties of the Koch monopole are more advantageous at this comparatively lower band since at frequencies like 2.4 and 5 GHz the wavelength is small enough to produce relatively small antennas. Dielectric losses are small at this frequency, thus it is possible to use a dielectric substrate which reduces electrical length without incurring noticeable radiation losses.

4.3 Fractal geometry

The geometric complexity required that the pattern be automatically generated. The geometry of the Koch fractal used in this design. The IFS algorithm described was implemented using the set of transforms specification to the Koch fractal given by (2.4). The script outputs the line vertex coordinates of Koch fractal of any given iteration. A scaling factor can also be given as a parameter to the script in order to generate coordinates for any physical size needed. A 3-iteration Koch fractal was generated to provide maximum height reduction. Although a 4- or higher-iteration fractal would be a further improvement. The Koch fractal was then scaled to have an equivalent unfolded length identical to the height of the straight-wire \( \left( \frac{4}{4} \right) \) monopole.
At 900 MHz, a straight-wire \( \frac{\lambda}{4} \) monopole has a height of \( h = 8.33 \) cm in free space.

According to expression (2.8), the equivalent Koch monopole would have a height of only 3.51 cm. However, it was determined in simulations that the electrical length was longer. This height is further reduced by the high dielectric constant of the substrate, as described in the next section. Through the geometry of fractals, the effective length of a Koch fractal antenna increases by \( \left( \frac{4}{3} \right)^n \) for each iteration where \( n \) is three numbers of iterations [3]. For an initial height \( h \), effective length for each antenna would be

\[
l = h \cdot (4/3)^n
\]  

(4.1)

<table>
<thead>
<tr>
<th>Antenna</th>
<th>Effective length</th>
</tr>
</thead>
<tbody>
<tr>
<td>k0</td>
<td>6.00 cm</td>
</tr>
<tr>
<td>k1</td>
<td>8.00 cm</td>
</tr>
<tr>
<td>k2</td>
<td>10.67 cm</td>
</tr>
<tr>
<td>k3</td>
<td>14.22 cm</td>
</tr>
</tbody>
</table>

**Table 4.1 Effective length of the Koch fractal antenna with a 6 cm physical length for various iterations**

Since every Koch fractal iteration increases the antenna length by 4/3, the length ratio can be estimated by where \( r \) is the scaling factor and \( \theta \) is the rotation angle. \( r = 1/3 \)

---

Figure 4.1- The Four Segments that form the basis of the Koch Fractal Antenna
Figure 4.2 Calculation of the distance $R$

$$h = \sqrt{\left(\frac{L_0}{2}\right)^2 + \left(\sin\left(\frac{\pi}{3}\right)\frac{L_0}{3}\right)^2} = 0.57735L_0, \quad r = \frac{R(n+1)}{R(n)} = \frac{57735L_0}{0.5L_0} = 1.155$$ (4.2)

Length scale ratio $\frac{4}{3} = 1.3333 \quad 1.3333 \geq r \geq 1.155, \quad r = 1.30 \text{ to } 1.25$

(a) k0
Figure 4.3- Koch Fractal of zero (a), one (b), two (c) and three (d) iterations [19]
4.4 Properties of Koch Fractal Monopole

Koch fractal monopole can improve the radiation property of a common linear monopole (resonance frequency, radiation resistance, bandwidths, radition Pattern) [11].

4.4.1 Resonant Frequency
A straight monopole, Koch fractal antennas with 1, 2 and 3 iterations. The effective height for each antenna is summarized in table 2. Diagrams of the four antenna are shown in fig4.3a,b,c,d. For each case the total physical height of the antenna was h=6 cm. The feed point for all of the antennas was located at the antenna / ground plane interface. The wire was considered to be a perfect conductor with a radius of 100um. When effective length of the antenna increases, the resonant frequency decreases

\[ F = \frac{c}{4l} \]  

Where \( c \) is the speed of light \( l \) is the effective length at that frequency, the fractal antenna should be similar to the quarter wave monopole.

4.4.2 Input impedance
Input impedance for linear dipole of length \( l \) and radius \( a \) can be [19]

\[ Z_{in} = 20\pi^2 \left( \frac{l}{\lambda} \right)^2 - j120 \left[ \frac{\ln\left( \frac{l}{2a} \right)}{\tan\left( \frac{\pi l}{\lambda} \right)} - 1 \right] \]  

\[ Z_{in} \text{ (monopole)} = \frac{1}{2} Z_{in} \text{ (dipole)} \]  

The input impedance for a small linear monopole can be found by dividing eq 4.4 by a factor of two. In table 4.2 for all three cases, the real part of the input impedance of the electrically short monopole is less than that for the Koch monopole. Also, for all three cases, the electrically short monopole has a significant negative reactive component, which becomes quite large at lower frequencies. The small real impedance and large negative reactance of the electrically short monopole can make the matching difficult.
Frequency | Input impedance for the Koch monopole | Input impedance for an electrically short monopole |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>983 MHz</td>
<td>23.1 Ω, k1</td>
<td>13.89 +j 134</td>
</tr>
<tr>
<td>836 MHz</td>
<td>17.0 Ω, k2</td>
<td>7.80 + j 266</td>
</tr>
<tr>
<td>740 MHz</td>
<td>13.4 Ω, k3</td>
<td>4.39 + j 415</td>
</tr>
</tbody>
</table>

Table 4.2 The Input Impedance of a 6 cm long Koch Fractal monopole and a 6 cm long electrically short monopole

4.4.3 Radiation Pattern

Straight monopole k0 and the three iteration fractal k3 the corresponding radiation pattern. It is interesting to note that for the E plane radiation pattern the fractal antenna also has a null at 90 degree. This is due to the symmetry of the fractal and the electric and magnetic fields cancelled in this direction. For the H plane the contours of the fractal antenna are pointing into the 0 degree direction. k3 radiates omni directionally due to the symmetry of the fractal antenna at resonance the electric and magnetic fields are added and cancelled in the far field to give a symmetrically pattern.

Figure 4.4- The E plane Radiation Pattern for the Straight monopole k0 and the fractal antenna k3 (Dashed =k0, solid= k3) [19]
4.4.4 Quality factor and bandwidth

Quality factor is used to measure the ratio of an antennas radiated energy to its stored energy. $Q$ of an antenna is inversely proportionally to its bandwidth. As the number of iteration of the fractal increases $Q$ approaches low value for small antenna. The limit for the smallest possible $Q$ for any antenna within the radian sphere, regardless of its shape, can be described as

$$Q = \frac{1 + 2(kr)^2}{(kr)^3 (1 + (kr)^2)} \quad (4.6)$$

The $Q$ of a small antennas may be described according to the stored electric and magnetic energies $W_e$ and $W_m$, respectfully, the frequency $\omega$ and the average radiated power $P_r$.

$$Q = \omega \frac{2W_e}{P_r}, W_e >> W_m \quad (4.7)$$

$$Q = \omega \frac{2W_m}{P_r}, W_m >> W_e \quad (4.8)$$

The stored electric and magnetic energies can be related to the input impedance of a lossless one port network with input reactance $X_{in}$ and susceptance $B_{in}$. The current and voltage at the input terminals of the antenna are $I$ and $V$, respectfully.
\[ W_e = \frac{1}{8} \left( \frac{\partial X_{in}}{\partial \omega} - \frac{X_{in}}{\omega} \right) = \frac{1}{8} \left( \frac{\partial B_{in}}{\partial \omega} - \frac{B_{in}}{\omega} \right) \] (4.9)

\[ W_m = \frac{1}{8} \left( \frac{\partial X_{in}}{\partial \omega} + \frac{X_{in}}{\omega} \right) = \frac{1}{8} \left( \frac{\partial B_{in}}{\partial \omega} + \frac{B_{in}}{\omega} \right) \] (4.10)

The power dissipated by the antenna can be described by

\[ P_L = \frac{1}{2} |I|^2 R_m = \frac{1}{2} |V|^2 G_m \] (4.11)

Putting eq 4.9, 4.10, 4.11 into equation 4.7 and 4.8, the following equations result:

\[ Q = \frac{\omega}{2R_m} \left( \frac{\partial X_{in}}{\partial \omega} - \frac{X_{in}}{\omega} \right), \quad W_e \gg W_m \] (4.12)

\[ Q = \frac{\omega}{2R_m} \left( \frac{\partial X_{in}}{\partial \omega} + \frac{X_{in}}{\omega} \right), \quad W_m \gg W_e \] (4.13)

Finally, the equivalent quality factor can be found by the following equation:

\[ Q = \frac{\omega}{2R_m} \left( \frac{\partial X_{in}}{\partial \omega} + \frac{X_{in}}{\omega} \right) \] (4.14)

So, that the fractal Koch monopole is actually quite low for a small antenna. As the number of iterations of the fractal increases.
4.4.5 Radiation Efficiency

When measuring the efficiency, the Wheeler cap method may be used. In this method, the total resistance is measured under two cases: the antenna is measured in free space, or the antenna is measured within a cylinder of metal, which is called a
Wheeler Cap. In free space, when electromagnetic waves encounter metal, it effectively looks as a short circuit and the waves are reflected. Therefore, when measuring the total resistance of an antenna with metal surrounding it, the radiation resistance is zero while the total resistance is comprised of only loss resistance. Formulas for the Wheeler Cap method are

\[
R_{\text{wc}} = R_{\text{loss}} \quad (4.15)
\]

\[
R_{\text{fs}} = R_{\text{loss}} + R_{\text{radius}} \quad (4.16)
\]

\[
R_{\text{radiated}} = R_{\text{fs}} - R_{\text{loss}} \quad (4.17)
\]

\[
\eta = \frac{R_{\text{Radiated}}}{R_{\text{Radiated}} + R_{\text{loss}}} \quad (4.18)
\]

Figure 4.8- Radiation efficiency measurement for various iterations of the Koch fractal antenna [19]

It was found that with each additional iteration of the fractal, the efficiency of the antenna slightly decreased. This is due to the fact that as the number of iterations increased, the radiation resistance decreased.
4.5 Hardware Design

4.5.1 Dielectric Substrate

As with the Sierpinski monopole design, the only available dielectric substrate is G10-FR4, with a relative dielectric constant $\varepsilon_r = 4.8$ and a thickness of 1.6 mm. However, at a frequency of 900 MHz, radiation losses are expected to be negligible. When electromagnetic waves propagate through a dielectric, they travel at a speed given by:

$$v = \frac{1}{\sqrt{\mu \varepsilon}}$$

Where $\varepsilon = \varepsilon_r \varepsilon_0$, since $\lambda = \frac{\sqrt{\varepsilon}}{f}$, the wavelength inside of the dielectric can then be expressed by:

$$\lambda = \frac{1}{\sqrt{(\mu \varepsilon, \varepsilon_0)}} \cdot \frac{1}{f}$$

(4.19)

This relationship is used to determine the required height of the monopole in order to radiate at 900 MHz while completely immersed in the FR-4 dielectric. This scaling effect is modeled using software simulations. Since the fractal pattern is essentially printed at the boundary between air and the dielectric substrate, the calculation of the required physical height of the Koch fractal is not as simple.

![Figure 4.9-900 MHz Koch monopole [20]](image)
Chapter-5
Simulation of Koch Fractal Antenna and Arrays

5.1 Introduction

In this chapter Koch fractal antennas and Arrays are designed and analyzed. A Koch monopole antenna is based on the Koch fractal curve. The simulated results of these designed antennas based on the respective fractal structures. Koch curve, Sierpinski Triangle, Sierpinski Carpet fractal were simulated using Matlab. Simulation results are shown below.

Figure 5.1 shows Koch curve for different iterations, 5.1(a) shows Koch curve for zero iteration, 5.1(b) Koch curve for one iteration, 5.1(c) Koch curve for two iteration, 5.1(d) Koch curve for three iteration, with each iteration the length of Koch increases by one third of its previous length. The Koch is constructed by adding smaller and smaller triangles to the structure in an iterative fashion.

Figure 5.1(a) - Koch curve with zero Iteration
Figure 5.1(b) - Koch curve with one iteration

Figure 5.1(c) - Koch curve with two iterations

Figure 5.1(d) - Koch curve with three iterations
Figure 5.2 shows Sierpinski triangle for different iterations, Fig 5.2 (a) shows Sierpinski triangle for zero iteration, Fig 5.2 (b) shows Sierpinski triangle for one iteration, Fig 5.2(c) shows Sierpinski triangle for two iteration. The procedure for geometrical construction of sierpinski gasket is explained in section 2.3.

Figure 5.2(a) - Sierpinski triangle for zero iteration

Figure 5.2(b) - Sierpinski triangle for one iteration
Figure 5.2(c) - Sierpinski triangles for two iteration.

Figure 5.3 shows Sierpinski Carpet for different iterations. Fig5.3 (a) shows Sierpinski Carpet for zero iteration, Fig5.3 (b) shows Sierpinski Carpet for one iteration, Fig5.3(c) shows Sierpinski Carpet for two iteration. The Sierpinski is constructed analogously to the Sierpinski Gasket but using solid squares instead of triangles. The interior of the center square is subtracted form the original square. From each of the remaining eight solid squares the centers of the squares are again removed. Repeating the process infinite number of times results in an ideal Sierpinski Carpet Fractal structure. Generation process is shown in Fig 5.3.

Figure5.3 (a) -Sierpinski Carpet for zero iteration
5.2 Koch Monopole Antenna

A Koch monopole antenna [20] is based on the Koch fractal curve. It is a simple multiband antenna intended to produce a much more space efficient quarter wave monopole and improve some of its features. This antenna behaves as an efficient radiator.

5.2.1 Antenna Geometry

The geometry of the antenna is described as under:

- The classical curve is generated by replacing the middle third of each straight section with a bent section of wire that spans the original third as shown in Figure 5.4.
IFS, as given in equation (2.6), are applied to succession of curves that converge to the ideal fractal shape. The affine transformation for this curve is formed by scaling factor of $r = 1/3$ and rotation angles of $\theta = 0^\circ, 60^\circ, -60^\circ$ and $0^\circ$. Thus there are four basic segments which form the basis of the Koch fractal curve, these are:

\[ w_1 = [1/3, \ 0, \ 0, \ 1/3, \ 0, \ 0] \quad (5.1) \]
\[ w_2 = [1/3\cos60, \ -1/3\sin60, \ 1/3\sin60, \ 1/3\cos60, \ 1/3, \ 0] \quad (5.2) \]
\[ w_3 = [1/3\cos60, \ 1/3\sin60, \ -1/3\sin60, \ 1/3\cos60, \ 1/2, \ 1/\sqrt{3}] \quad (5.3) \]
\[ w_4 = [1/3, \ 0, \ 0, \ 1/3, \ 2/3, \ 0] \quad (5.4) \]

![Figure 5.4. Four segments of Koch curve](image)

It has been observed that the effective length of a Koch fractal antenna increases by $(4/3)^n$ for each iteration, where $n$ is the number of iterations. This means for an initial height $h$ effective length for each antenna is given by:

\[ l = h.(4/3)^n \quad (5.5) \]
Table 5.1: Effective length of Koch monopole of physical height 2.1 cm

<table>
<thead>
<tr>
<th>Antenna</th>
<th>Effective length</th>
</tr>
</thead>
<tbody>
<tr>
<td>k0</td>
<td>2.1 cm</td>
</tr>
<tr>
<td>k1</td>
<td>2.8 cm</td>
</tr>
<tr>
<td>k2</td>
<td>3.733 cm</td>
</tr>
</tbody>
</table>

5.3 Koch Fractal Antenna for GSM 900

A Koch monopole antenna of 2.1 cm with Second iterations (k2) is considered over a ground plane. The feed point for the antennas is located at the antenna/ground plane interface as shown in Fig 5.5

GSM900 operates at frequency range 890-915Mhz. for uplink communication and 935-960Mhz. for downlink communication. A monopole on a perfect ground having resonance at 925 MHz is required. By using a two iteration Koch, the length of Koch monopole required is 2.1 cm.(from equation 5.5)to provide effective height 3.73 cm., second iteration on a perfect ground with source at bottom end is used. Radius of wire has been taken 0.1mm. With radius 0.1mm the antenna has bandwidth (SWR<2) 25 MHz, which is very less to cover 900 MHz band, by increasing wire radius, bandwidth could be increased. By taking radius 3.6mm. Bandwidth (SWR<2) increases up to 70Mhz. which covers the whole 900 MHz band, and provides a gain of 4.9db.

5.3.1 SIMULATION RESULTS:

Using Matlab a Koch with two iterations 2.1cm long has been generated and using MMANA code which is a MININEC code, antenna is simulated. The Koch monopole exhibits excellent performance at 925 MHz and has radiation properties nearly identical to that of traditional, straight-wire monopoles at that frequency. The radiation pattern is very uniform in all directions. Simulation results are shown below.
Figure 5.5 - Koch of length 2.1 cm of two iterations with source at bottom on a perfect ground of wire radius 0.1 mm.

5.3.1.1 Input Impedances:
The Input Impedance (real and imaginary part) for Koch monopole antennas on GSM 900 of physical heights 2.1 cm of Second iterations with resonant frequencies 925 MHz are shown in Figure 5.6. It shows the impedance decreases as radius of wire increases.
(b) Radius 2 mm

(c) Radius 3 mm
(d) Radius 3.6 mm

Figure 5.6- Plot of Frequency versus Real and Imaginary part of impedance for varying Radius at GSM 900 MHz.

5.3.1.2 Standing Wave Ratio:
The Standing wave Ratio for Koch monopole antennas on GSM 900 of physical heights 2.1 cm of Second iterations with resonant frequencies 925 MHz are shown in Figure 5.7.
(b) Radius 2 mm

(c) Radius 3 mm
5.3.1.3 Gain and Front to Back Ratio

The Gain and Front Back Ratio for Koch monopole antennas on GSM 900 of physical heights 2.1 cm of two iterations with resonant frequencies 925 MHz are shown in Figure 5.8. Gain and front back ratio remains constant as wire radius increases.
(b) Radius 2mm

(c) Radius 3 mm
5.3.1.4 Radiation Pattern:
The Radiation Pattern for Koch monopole antennas on GSM 900 of physical height 2.1 cm of two iterations with resonant frequencies 925 MHz are shown in Figure 5.9. It radiates omni directionally for all radius.

Elevation Plot for Radius 0.1 mm
Azimuthal Plot for Radius 0.1 mm

Elevation Plot for Radius 2 mm

Azimuthal plot for Radius 2 mm

Elevation Plot for Radius 3 mm
Figure 5.9-Plot of Radiation Pattern of Koch Antenna for varying radius at GSM 900 MHz
5.3.1.5 Quality Factor:
The Quality Factor for Koch monopole antennas on GSM 900 of physical height 2.1cm of two iterations with resonant frequencies 925 MHz are shown in Figure 5.10. Quality factor decreases as wire radius increases.

![Figure 5.10. Quality Factor vs. Radius (mm) of Koch Antenna at GSM 900 MHz](image)

5.3.1.6 Bandwidth:
The Bandwidth for Koch monopole antennas on GSM 900 of physical height 2.1cm of two iterations with resonant frequencies 925 MHz are shown in Figure 5.11. Bandwidth increases as wire radius increases.

![Figure 5.11- Bandwidth vs. Radius (mm) of Koch Antenna at GSM 900 MHz](image)
5.3.2 Results and Discussions

The various observations made from the results obtained for antenna: Koch Monopole Fractal Antenna on GSM 900 is discussed below:

5.3.2.1 Koch Monopole Antenna

The various parameters of the Koch monopole antenna of Second iteration of 2.1 cm physical length are listed in table 5.2.

<table>
<thead>
<tr>
<th>Radius (mm)</th>
<th>Real part of impedance (Ω)</th>
<th>Imaginary part of impedance (Ω)</th>
<th>Gain (dB)</th>
<th>Front and back ratio</th>
<th>Quality factor</th>
<th>Bandwidth (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>9.266</td>
<td>-129.862</td>
<td>4.88</td>
<td>-0.02</td>
<td>22.2</td>
<td>25</td>
</tr>
<tr>
<td>2.0</td>
<td>8.326</td>
<td>-78.035</td>
<td>4.88</td>
<td>-0.01</td>
<td>10.5</td>
<td>53</td>
</tr>
<tr>
<td>3.0</td>
<td>8.040</td>
<td>-66.660</td>
<td>4.90</td>
<td>-0.01</td>
<td>8.9</td>
<td>64</td>
</tr>
<tr>
<td>3.6</td>
<td>7.922</td>
<td>-61.373</td>
<td>4.90</td>
<td>-0.01</td>
<td>8.2</td>
<td>70</td>
</tr>
</tbody>
</table>

- As wire radius increasing and quality factor decreasing the bandwidth starts increases for a radius of 3.6 mm Bandwidth goes up to 70 MHz which is required by GSM 900 using second iteration Koch Fractal antennas of 2.1 cm
- Input impedance decreases with increasing wire radius.
- Other factors like Front and back Ratio and Gain Remains almost constant.
- Antenna radiates omni directionally at all radius of wire. The E-plane radiation shows a null at 90°. This is due to the symmetry of the fractal and the electric and magnetic field cancelled in this direction. The H-
plane the contours of fractal antennas points to the $0^\circ$ direction. This shows that fractal antenna radiates omni directionally. This is due to the symmetry of fractal antenna at resonance the electric and magnetic fields are added and canceled in the far field and give a symmetrical pattern.

5.4 Koch Fractal Antenna Array for GSM 900

GSM 900 operates at frequency range 890-915MHz for uplink communication and 935-960 MHz for downlink communication. A monopole on a perfect ground having resonance at 925 MHz is required. According to linear antenna array second iteration three elements Koch monopole of equally spaced elements (0.02m) with voltage of equal magnitude with uniform progressive phase shift along the line is shown in Fig 5.12. Three elements Koch of length 2.1 cm. with two iteration on a perfect ground with source at bottom end are used. Radius of wire has been taken 0.1mm. With radius 0.1mm antenna has (SWR<2) bandwidth 71 MHz, by increasing wire radius, bandwidth increases. By taking radius 3.6mm. Bandwidth increases up to 155MHz and provide a gain of 6.78 dB.

![Fig 5.12- Three Element Koch Fractal Array of 2.1 cm of two Iteration](image)
5.4.1. SIMULATION RESULTS:
Using Matlab a Koch with two iterations 2.1cm long has been generated and using MMANA code which is a MININEC code, antenna is simulated. Simulation results are shown below.

5.4.1.1 Input Impedances:
The Input Impedance (real and imaginary part) for Koch monopole antennas arrays on GSM 900 of physical height 2.1 cm of Second iterations with resonant frequencies 925 MHz are shown in Figure 5.13. It shows the impedance decreases as radius of wire increases.

(a) Radius 0.1 mm

(b) Radius 2 mm
Figure 5.13- Plot of Frequency versus Real and Imaginary part of impedance for varying radius of Koch Antenna Array at GSM 900 MHz.

5.4.1.2 Standing Wave Ratio

The Standing wave Ratio for Koch monopole antennas arrays on GSM 900 of physical height 2.1 cm of Second iterations with resonant frequencies 925 MHz are shown in Figure 5.14.
(a) Radius 0.1 mm

(b) Radius 2 mm
Figure 5.14- Plot of Frequency versus SWR for varying radius of Koch Antenna Array at GSM 900 MHz
5.4.1.3 Gain and Front to Back Ratio

The Gain and Front Back Ratio for Koch monopole antennas arrays on GSM 900 of physical height 2.1 cm of two iterations with resonant frequencies 925 MHz are shown in Figure 5.15. Gain and front back ratio remains constant as wire radius increases.
Figure 5.15- Plot of Frequency versus Gain and Front to back ratio for varying radius of Koch Antenna Array at GSM 900 MHz

5.4.1.4 Radiation Pattern:
The Radiation Pattern for Koch monopole antennas arrays on GSM 900 of physical height 2.1cm of two iterations with resonant frequencies 925 MHz are shown in Figure 5.16. Its Radiation Pattern slightly changes from Koch antenna.
Elevation Plot for Radius 2 mm

Azimuthal Plot for Radius 2 mm

Elevation Plot for Radius 3 mm
Figure 5.16- Radiation Pattern of Koch Antenna Array for varying Radius at GSM 900 MHz
5.4.1.5 Quality Factor:
The Quality Factor for Koch monopole antennas arrays on GSM 900 of physical height 2.1cm of two iterations with resonant frequencies 900 MHz are shown in Figure 5.17. Quality factor decreases as wire radius increases.

![Figure 5.17- Quality Factor vs. Radius (mm) for Koch Antenna Array at GSM 900 MHz](image1.png)

5.4.1.6 Bandwidth:
The Bandwidth for Koch monopole antennas arrays on GSM 9000 of physical height 2.1cm of two iterations with resonant frequencies 925 MHz are shown in Figure 5.18. Bandwidth increases as wire radius increases.

![Figure 5.18- Bandwidth vs. Radius (mm) for Koch Antenna Array at GSM 900 MHz](image2.png)
5.4.2 Results and Discussions

The various observations made from the results obtained for antenna: Koch arrays, Koch Fractal Antenna Array on GSM 900 are discussed below:

5.4.2.1 Koch Monopole Antenna Arrays

The various parameters of the Koch monopole arrays of Second iteration of 2.1 cm physical length are listed in table 5.3

Table 5.3 Koch fractal antenna array for GSM 900

<table>
<thead>
<tr>
<th>Radius (mm)</th>
<th>Real part of impedance (Ω)</th>
<th>Imaginary part of impedance (Ω)</th>
<th>Gain (dB)</th>
<th>Front and back ratio</th>
<th>Quality factor</th>
<th>Bandwidth (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>22.694</td>
<td>-113.029</td>
<td>6.64</td>
<td>3.04</td>
<td>9.0</td>
<td>71</td>
</tr>
<tr>
<td>2.0</td>
<td>15.900</td>
<td>-59.300</td>
<td>6.69</td>
<td>3.13</td>
<td>4.9</td>
<td>130</td>
</tr>
<tr>
<td>3.0</td>
<td>13.964</td>
<td>-47.819</td>
<td>6.74</td>
<td>3.23</td>
<td>4.4</td>
<td>147</td>
</tr>
<tr>
<td>3.6</td>
<td>13.075</td>
<td>-42.584</td>
<td>6.78</td>
<td>3.31</td>
<td>4.2</td>
<td>155</td>
</tr>
</tbody>
</table>

- Bandwidth Increases up to 155MHz using three element Koch Fractal antennas of 2.1 cm for GSM 900 by decreasing the quality factor and increasing the radius of wire. It gives 70 MHz Bandwidth for radius 3.6mm in case of single element Koch fractal whereas Koch array gives 71 MHz bandwidth at radius 0.1 mm.
- Input impedance decreases as increasing the wire radius. But input impedance increases using array as compared to single element Koch.
- Other factors like Front and back Ratio and Gain Remains almost constant for all radius. But gain and directivity also increases using array as compared to single element. Koch array gives 6.78 (dB) gain and single Koch gives 4.9 (dB) gain.
- The E-plane radiation shows a null at 90°. This is due to the symmetry of the fractal and the electric and magnetic field cancelled in this direction. Elevation plot remains almost omnidirection and in case of azimuthal
plot there is a slightly change in distribution of energy in both lobes for Koch array.

5.5 Koch Fractal Antenna for GSM 1800

GSM1800 operates at frequency range 1710-1785 MHz for uplink communication and 1805-1880MHz for downlink communication. A monopole on a perfect ground having resonance at 1800 MHz is required. By using a two iteration Koch, the length of Koch monopole required is 2.1cm (from equation 5.5) to provide effective height of 3.73cm., Koch of length 2.1 cm. with two iteration on a perfect ground with source at bottom end is used as shown in Fig 5.19. Radius of wire has been taken 0.001mm. With radius 0.001mm antenna has (SWR<2) bandwidth 73 MHz which is very less to cover 1800 MHz band, by increasing wire radius, bandwidth increases. By taking radius 0.16mm Bandwidth increases up to 180MHz. which covers the whole 1800 MHz band and provide a gain of 5.32 dB.

5.5.1. Simulation Results

Using Matlab a Koch of two iterations on height 2.1cm has been generated and using MMANA code which is a MININEC code, antenna is simulated. The Koch monopole exhibits excellent performance at 1800 MHz and has radiation properties nearly identical to that of traditional, straight-wire monopoles at that frequency. Simulation results are shown below.

Figure 5.19- Koch of length 2.1cm. of two iterations with source at bottom on a perfect ground of wire radius 0.001mm.
5.5.1.1 Input Impedances:
The Input Impedance (real and imaginary part) for Koch monopole antenna on GSM 1800 of physical height 2.1 cm of Second iterations with resonant frequencies 1800 MHz are shown in Figure 5.20. It shows the impedance decreases as radius of wire increases.
Figure 5.20 - Plot of Frequency versus Real and Imaginary part of impedance for varying Radius of Koch Antenna at GSM 1800 MHz
5.5.1.2 Standing Wave Ratio:

The Standing wave Ratio for Koch monopole antenna on GSM 1800 of physical height 2.1 cm of Second iterations with resonant frequencies 1800 MHz are shown in Figure 5.21.

(a) Radius .001 mm

(b) Radius 0.12 mm
Figure 5.21. Plot of Frequency vs SWR for varying Radius of Koch Antenna at GSM 1800 MHz

5.5.1.3 Gain and Front to Back Ratio
The Gain and Front Back Ratio for Koch monopole antenna on GSM 1800 of physical height 2.1 cm of two iterations with resonant frequencies 1800 MHz are shown in Figure 5.22. Gain and front back ratio remains almost constant as wire radius increases.
Figure 5.22-Plot of Frequency versus Gain and Front to back ratio for varying
Radius of Koch Antenna at GSM 1800 MHz

(c) Radius 0.14 mm

(d) Radius 0.16 mm
5.5.1.4 Radiation Pattern:

The Radiation Pattern for Koch monopole antenna on GSM 1800 of physical height 2.1 cm of two iterations with resonant frequencies 1800 MHz are shown in Figure 5.23. It Radiate omni directionally for all radius.

Elevation Plot at Radius 0.001 mm

Azimuthal plot at Radius 0.001 mm
Elevation Plot at Radius 0.12 mm

Azimuthal plot at Radius 0.12 mm

Elevation Plot at Radius 0.14 mm
Figure 5.23-Plot of Radiation Pattern for Koch Antenna for varying radius at GSM 1800 MHz
5.5.1.5 Quality Factor:

The Quality Factor for Koch monopole antenna on GSM 1800 of physical height 2.1cm of two iterations with resonant frequencies 1800 MHz are shown in Figure 5.24. Quality factor decreases as wire radius increases.

![Figure 5.24 - Quality Factor vs radius (mm) for Koch Antenna at GSM 1800 MHz](image)

5.5.1.6 Bandwidth:

The Bandwidth for Koch monopole antenna on GSM 1800 of physical heights 2.1cm of two iterations with resonant frequencies 1800 MHz are shown in Figure 5.25. Bandwidth increases as wire radius increases.

![Figure 5.25 - Bandwidth vs. radius (mm) for Koch Antenna at GSM 1800 MHz](image)
5.5.2 Results and Discussions

The various observations made from the results obtained for antenna: Koch Monopole Fractal Antenna on GSM 1800 is discussed below:

5.5.2.1 Koch Monopole Antenna

The various parameters of the Koch monopole antennas of 2.1 cm physical length are listed in Table 5.4

Table 5.4. Koch fractal antenna for GSM 1800

<table>
<thead>
<tr>
<th>Radius (mm)</th>
<th>Real part of impedance (Ω)</th>
<th>Imaginary part of impedance (Ω)</th>
<th>Gain (dB)</th>
<th>Front and back ratio</th>
<th>Quality factor</th>
<th>Bandwidth (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>106.160</td>
<td>640.234</td>
<td>5.32</td>
<td>-0.06</td>
<td>14</td>
<td>73</td>
</tr>
<tr>
<td>0.12</td>
<td>92.014</td>
<td>257.382</td>
<td>5.32</td>
<td>-0.04</td>
<td>6.2</td>
<td>168</td>
</tr>
<tr>
<td>0.14</td>
<td>91.346</td>
<td>245.579</td>
<td>5.32</td>
<td>-0.04</td>
<td>5.9</td>
<td>174</td>
</tr>
<tr>
<td>0.16</td>
<td>90.765</td>
<td>230.847</td>
<td>5.32</td>
<td>-0.04</td>
<td>5.7</td>
<td>180</td>
</tr>
</tbody>
</table>

- As wire radius increasing and quality factor decreasing the Bandwidth starts increases for a radius of 0.16 mm the Bandwidth goes up to 180 MHz which is required by GSM 1800 using second iteration Koch Fractal antennas of 2.1 cm
- Input impedance decreases as increasing the wire radius.
- Other factors like Front and back Ratio and Gain Remains almost constant. It gives 5.32 (dB) gain.
- Antenna radiates omni directionally at all radius of wire for Koch antenna.

5.6 Koch Fractal Antenna Array for GSM 1800
GSM1800 operates at frequency range 1710-1785 MHz for uplink communication and 1805-1880 MHz for downlink communication. A monopole on a perfect ground having resonance at 1800 MHz is required. According linear antenna array second iteration three element Koch monopole of equally spaced elements (0.02m) with voltage of equal magnitude with uniform progressive phase shift along the line as shown in Fig 5.26. Three elements Koch of length 2.1 cm. with two iteration on a perfect ground with source at bottom end are used as shown in Fig 5.26. Radius of wire has been taken 0.001mm. With radius 0.001mm antenna has (SWR<2) bandwidth 81 MHz which is very less to cover 1800 MHz band, by increasing wire radius, bandwidth increases. By taking radius 0.12mm Bandwidth increases up to 182 MHz. which covers the whole 1800 MHz band and provide a gain of 8.05 dB.

5.6.1 Simulation Results:
Using Matlab a Koch of two iterations on height 2.1 cm. Has been generated and using MMANA antenna is simulated.
5.6.1.1 Input Impedances:

The Input Impedance (real and imaginary part) for Koch monopole antenna arrays on GSM 1800 of physical height 2.1 cm of Second iterations with resonant frequencies 1800 MHz are shown in Figure 5.27. It shows the impedance decreases as radius of wire increases.

(a) Radius .001 mm

(b) Radius 0.12 mm
Figure 5.27-Plot of Frequency versus Real and Imaginary part of impedance for varying radius of Koch array at GSM 1800 MHz

5.6.1.2 Standing Wave Ratio:
The Standing wave Ratio for Koch monopole antennas array on GSM 1800 of physical height 2.1 cm of Second iterations with resonant frequencies 1800 MHz are shown in figure 5.28. As wire radius increases.

(a) Radius 0.001 mm

(b) Radius 0.12 mm
Figure 5.28 - Plot of Frequency versus SWR for varying radius of Koch array at GSM 1800 MHz

5.6.1.3 Gain and Front to Back Ratio
The Gain and Front Back Ratio for Koch monopole antenna arrays on GSM 1800 of physical height 2.1 cm of two iterations with resonant frequencies 1800 MHz are shown in Figure 5.29. Gain and front back ratio remains almost constant as wire radius increases.
5.6.1.4 Radiation Pattern:

The Radiation Pattern for Koch monopole antenna arrays on GSM 1800 of physical height 2.1cm of two iterations with resonant frequencies 1800 MHz are shown in Figure 5.30. Its Radiation Pattern slightly changes from Koch antenna.
Elevation Plot at Radius 0.001 mm

Azimuthal plot at radius 0.001 mm

Elevation Plot at Radius 0.12 mm
Azimuthal plot at Radius 0.12 mm

Elevation Plot at Radius 0.14 mm

Azimuthal plot at Radius 0.14 mm
Elevation Plot at Radius 0.16 mm

Azimuthal plot at Radius 0.16 mm

Figure 5.30- Plot of Radiation Pattern of Koch Array for varying radius at GSM 1800 MHz

5.6.1.5 Quality Factor:

The Quality Factor for Koch monopole antenna arrays on GSM 1800 of physical height 2.1cm of two iterations with resonant frequencies 1800 MHz are shown in Figure 5.31. Quality factor decreases as wire radius increases.
5.6.1.6 Bandwidth:

The Bandwidth for Koch monopole antenna arrays on GSM 1800 of physical height 2.1cm of two iterations with resonant frequencies 1800 MHz are shown in Figure 5.32. Bandwidth increases as wire radius increases.
5.6.2 Results and Discussions

The various observations made from the results obtained for antennas: Koch arrays, Koch Fractal Antenna Arrays on GSM 1800 is discussed below:

5.6.2.1 Koch Monopole Antenna Arrays

The various parameters of the Koch monopole antenna Arrays of second iteration of 2.1 cm physical length are listed in table 5.5

Table 5.5. Koch fractal antenna array for GSM 1800

<table>
<thead>
<tr>
<th>Radius (mm)</th>
<th>Real part of impedance (Ω)</th>
<th>Imaginary part of impedance (Ω)</th>
<th>Gain (dB)</th>
<th>Front and back ratio</th>
<th>Quality factor</th>
<th>Bandwidth (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>122.957</td>
<td>624.603</td>
<td>7.82</td>
<td>-8.23</td>
<td>12.9</td>
<td>81</td>
</tr>
<tr>
<td>0.12</td>
<td>94.011</td>
<td>224.586</td>
<td>8.02</td>
<td>-8.47</td>
<td>5.6</td>
<td>182</td>
</tr>
<tr>
<td>0.14</td>
<td>92.648</td>
<td>212.999</td>
<td>8.04</td>
<td>-8.47</td>
<td>5.4</td>
<td>189</td>
</tr>
<tr>
<td>0.16</td>
<td>91.450</td>
<td>203.106</td>
<td>8.05</td>
<td>-8.48</td>
<td>5.2</td>
<td>195</td>
</tr>
</tbody>
</table>

- Bandwidth Increases very fastly up to 182 MHz using three elements Koch fractal array of 2.1 cm with second iteration for GSM 1800 at Radius 0.12 as compared to single element Koch fractal on GSM 1800. it gives 180 MHz bandwidth at radius 0.16mm using single element Koch fractal whereas Koch array gives 195 MHz bandwidth at radius 0.16mm.
- Input impedance increases using array as compared to single element Koch fractal.
- Gain and Directivity also increases using array as compared to single element Koch fractal. Koch array gives 8.05 (dB) gain and single Koch gives 5.32 (dB) gain.
The E-plane radiation shows a null at 90°. This is due to the symmetry of the fractal and the electric and magnetic field cancelled in this direction. Elevation plot remains almost omnidirection and in case of azimuthal plot there is a slightly change in distribution of energy in both lobes for Koch array.
Chapter 6
Conclusions and Future Scope

In this dissertation, Koch Fractal Antenna and Koch Fractal antenna Arrays incorporated into GSM handsets have been purposed. The thesis involves simulation of Koch fractal antennas. Koch fractal antennas have been simulated using MATLAB and MMANA codes. In chapter-5 results shows that Koch fractal monopole are an excellent alternative to traditional antenna systems in mobile wireless receivers The Koch monopole exhibits excellent performance at 925 MHz and 1800MHz and has radiation properties nearly identical to that of traditional, straight-wire monopoles at that frequency. The radiation pattern is very uniform in all directions. The greatest advantage of the Koch monopole design is compactness. This is highly significant for applications such as GSM cellular phones. Since it is half the size of the traditional monopole, it could easily be completely integrated within the case of the phone. The Koch monopole design has excellent impedance bandwidth, allowing some flexibility in the types of applications where it could be used. Since the radiation pattern is highly uniform and identical to that of a traditional $\lambda/4$ monopole, it could be used in nearly any type of wireless communications receiver.

In Chapter-5 Three elements Koch fractal antenna array of 2.1 cm of two iteration on GSM gives excellent performances as compared to one element Koch. When three elements Koch array on GSM 900 compared with single Koch. It is found that Koch array improves the gain, directivity and impedance. It gives the bandwidth 155 MHz at radius 3.6 mm whereas single Koch gives the bandwidth 70 MHz at radius 3.6 mm and Koch array covers 70 MHz bandwidth at radius 0.1mm.

Three elements Koch fractal array on GSM 1800 covers Bandwidth is 180 MHz at radius 0.12mm and 195 MHz bandwidth at radius 0.16mm. Whereas single element Koch covers the bandwidth 180 MHz at radius 0.16(mm). Array method also increases the gain, directivity, input impedances but slightly change
in its radiation pattern. Thus the three elements Koch monopole antenna array presents an excellent, compact solution over one element Koch antenna on GSM 900, 1800.

**Future Scope**

Since the area of fractal antenna engineering research is still in its infancy, there are many possibilities for future work on this topic. The Koch fractal was chosen for this project because this is the best documented fractal antenna types in current research. However, many possible fractal structures exist which may undoubtedly have desirable radiation properties. Thus, a possible approach for future work is to investigate other types of fractals for antenna applications. A novel development is the use of fractal patterns for antenna arrays. Fractal antennas can be studied in several areas. One area of development is to implement fractal antennas into current technologies in practical situations such as expanding wireless market. For this application an analysis of the polarization of these antennas will need to be looked. Another benefit that can be explored is lower covered area of resonant loop antennas. This may lead to antenna with lower cross sections. Also, fractals can be used into micro strip antennas. Practical implementation of the proposed Koch can be taken up for farther study and all the simulated parameters can be verified practically also.
References


[27] www.Fractals in nature & applications.htm