Thesis report

on

STUDY AND ANALYSIS OF TIMING OFFSET ESTIMATION METHODS IN OFDM SYSTEMS

Thesis submitted in partial fulfilment of 
the requirement for the award of degree of

MASTER OF ENGINEERING

IN

WIRELESS COMMUNICATION

Submitted by

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DECLARATION

I hereby declare that the work, which is being presented in the thesis, entitled “STUDY AND ANALYSIS OF TIMING OFFSET ESTIMATION METHODS IN OFDM SYSTEMS” in partial fulfilment of the requirements for the award of Master of Engineering in Wireless Communication at Electronics and Communication Engineering Department of Thapar University, Patiala which is an authentic record of my own work carried out under the guidance of Dr. Hem Dutt Joshi (Assistant Professor), Electronics and Communication Engineering Department during the year 2012-2014.

The content in this thesis has not been submitted to any other university for the award of any other degree.

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Deepak Bhandari
ABSTRACT

OFDM is a multi-carrier transmission technique. It converts the frequency selective fading channel into a collection of parallel flat fading sub-channels which simplifies the structure of a receiver. OFDM has been widely used in communication system and is the key technology of 4G in the field of mobile communication because of its excellent performance in overcoming multipath channel and narrow band interference. However, wireless environment causes ISI and ICI. Synchronization is one of the major problems that affect the system performance in OFDM system. We basically focus on symbol timing. Finding the symbol timing for OFDM means finding an estimate of where the symbol starts. OFDM requires timing synchronization to preserve the orthogonality. The receiver will obtain the timing information from the received OFDM signal, but the information obtained comprises of offsets as well as the estimation errors. Synchronization techniques consists of data aided and non-data aided. Data aided makes the use of additional data which consists of pilot and preamble. Non data aided make the use of correlation between the CP and the end of the Symbols. The correction of timing and frequency offset plays the significant role in OFDM system. Estimating the starting point of an OFDM system must be handled efficiently and effectively to reduce errors.

In data aided technique, the most common synchronization technique is the Schmidl and Cox which has been followed by different researchers trying to get the better results based on this algorithm. Schmidl and Cox used a special structure having the two symbol preamble to estimate the timing offsets. They constructed the first training symbol with two identical halves but this technique shows the disadvantage in large plateau. To overcome these problems, Minn proposed a new training sequence consisting of four symbols which uses the concept of negative samples. Plateau uncertainty is removed. Park also modified the preamble taking the reference of Minn’s algorithm in order to increase the sharpness of peaks but still had some problem occurring of to many side lobes. The problem faced in park’s algorithm is removed by K. Shi and E. Serpedin algorithm but this algorithm also has the main problem on complexity which produces the better peak than that of Schmidl and Minn but not that of park. Many of the methods were investigated and analysed. We have proposed a new timing synchronization based on CP STO estimation technique and the training symbol based which helps in finding the performance of STO estimation. Correlation technique is affected when using CFO as well as the minimum difference is not affected when adding CFO when simulating.
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<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
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<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time Division Multiple Access</td>
</tr>
<tr>
<td>B-ISDN</td>
<td>Broadband Integrated Services Digital Network</td>
</tr>
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<td>IMT</td>
<td>International Mobile Telecommunication</td>
</tr>
<tr>
<td>DS-CDMA</td>
<td>Direct Sequence Code Division Multiple Access</td>
</tr>
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<td>MS-CDMA</td>
<td>Multi-carrier Code Division Multiple Access</td>
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<td>FDD</td>
<td>Frequency Division Duplexing</td>
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<td>RMS</td>
<td>Root Mean Square</td>
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<td>WLAN</td>
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<td>FHSS</td>
<td>Frequency Hoping Spread Spectrum</td>
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<td>FEC</td>
<td>Forward Error Correction Coding</td>
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<td>QAM</td>
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<td>Differential Quadrature Phase Shift Keying</td>
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<td>ICI</td>
<td>Inter Carrier Interference</td>
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<td>CP</td>
<td>Cyclic Prefix</td>
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<td>ISI</td>
<td>Inter Symbol Interference</td>
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<td>LTI</td>
<td>Linear Time Invariant</td>
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<td>MAC</td>
<td>Medium Access Control</td>
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<td>ADSL</td>
<td>Asymmetric Digital Subscriber Line</td>
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<tr>
<td>BWA</td>
<td>Broadband Wireless Access</td>
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<tr>
<td>DAB</td>
<td>Digital Audio Broadcasting</td>
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<td>DVB</td>
<td>Digital Video Broadcasting</td>
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<td>HIPERLAN</td>
<td>High Performance Local Area Network</td>
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<tr>
<td>IDFT</td>
<td>Inverse Discrete Fourier Transform</td>
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<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
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<tr>
<td>IFFT</td>
<td>Inverse Fast Fourier Transform</td>
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<td>MAN</td>
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<td>CIR</td>
<td>Channel Impulse Response</td>
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<td>STO</td>
<td>Symbol Timing Offset</td>
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<td>ML</td>
<td>Maximum Likelihood</td>
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<td>4G</td>
<td>Fourth Generation</td>
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CHAPTER 1

INTRODUCTION

1.1 OUTLINE

The platform of modern communication system can be called as the multi-carrier transmission. A special multi-carrier transmission technique used in WLAN system is OFDM, where a lot of rapid development has been occurred up to this phase. The OFDM has been adopted by many wireless communications standards like IEEE 802.11 and IEEE 802.16 family.

Analog cellular technologies which use the FM modulation refer to 1G was introduced in the year 1980s. It operates on the frequency band of 800 MHz-900MHz which uses the FDMA technology to divide the total system bandwidth into specific frequency channels [1, 2]. Because of the different frequencies and the different protocols used, first generation systems were unreliable which leads to the problem of roaming so that the user had to change their mobile terminals when travelling from one place to another. When a mobile’s user phone is activated, it is registered in that specific location. When the user roams into a new specified area covered by different service provider, the wireless network must register the user in the new location and cancels its registration with the new service provider [3]. To overcome from these problem, second generation mobile standards were evolved.

Two types of 2G system standardization was deployed in 1990s. One is deployed originally in Europe in the name of GSM which is still relevant in today’s context. The other is the PCS deployed originally in USA. The use of digital voice coding, digital modulation and advanced call processing capabilities are basically used in second generation system. 2G provides the digital data services in the form of short message and the voice over the mobile radio channel. 2G system is basically designed to reduce the computational and switching burden at the base station or the mobile switching centre while allocating the channel scheme for providing more flexibility. The successful development of 2G enhanced the 2.5G standards to overcome its problem occurred in the past decades [4]. Second generation can provide the large number of user with the reliable amount of data services with roaming but this generation has still some problem in providing the high speed internet and multimedia services [3]. Although the development of 2G and an enhanced 2.5G, these generation were not able to provide the high quality
service in different sectors. So to minimise the problems third generation mobile standards were developed.

The standardization of the 3G system was commenced in late 1990s. Depending on the mobility and location, the main 3G standards which are WCDMA standard and UMTS provides the different data rates. The use of broadband integrated services digital network (B-ISDN) to provide the access to the information network, likewise in internet, public and private databases are the salient features of third generation wireless networks. Third generation standardization allows the network operators to use in a wide range the advanced services by a user having the greater capacity through its enhanced spectral efficiency. On the basis of CDMA standardized by IMT-2000, three radio access schemes has been proposed as direct sequence CDMA (DS-CDMA)-frequency division duplexing(FDD), multi-carrier CDMA(MC-CDMA)-FDD and DS-CDMA- time division duplexing(TDD). There are basically two major 3G standards developed by IMT-2000 as WCDMA and CDMA-2000. The data rates for WCDMA and CDMA-2000 are 144 Kbps for high mobility, 384 Kbps for low mobility and 2Mbps for stationary which will be applicable for both standards [2]. They both have the frequency band of 2GHz band. WCDMA must have at least the spectrum allocation of 5MHz.

The final step towards the 4G system is LTE which has been classified as pre-4G by IMT. In the present context, wireless communication is jumping towards the next generation which has been characterized by all IP based networks. Another area for which the intense research has been occurred is the fourth generation mobile wireless technologies whose main focus is in the spectral efficiency. OFDM based techniques and the OFDM are the most promising techniques in terms of spectral efficiency. 4G system will be totally IP-based integrated system of system and networks of networks which will be capable of providing higher quality of service and security at the speed rate of 100 Mbps and 1Gbps respectively in indoor and outdoor environments so that it will be feasible to provide any type of services at affordable time [5].

1.2 MULTIPLE ACCESS TECHNIQUES [3]

The technique to allow the mobile users to share a common medium simultaneously in an effective and in efficient manner can be defined as multiple accesses. The major types of multiple access techniques are FDMA, TDMA and CDMA. Depending on the available
bandwidth allocated to the user, multiple access techniques can be differentiated into narrowband and wideband system which are briefly discussed below.

**Narrowband system**

The user in the narrowband FDMA assigns a particular channel which is not being shared by other users in the immediate surrounding area. If the system uses the FDD then it is called FDMA/FDD. The user in the narrowband TDMA shares the same channel which uses a unique time slot in a cyclical order. The allocations of large number of channels are used using either FDD or TDD which can be called it as TDMA/FDD or TDMA/TDD.

**Wideband system**

This system allows the user to transmit in the wide part of the spectrum and the greater number of spectrum which is both allowed to transmit on the same channel. The allocation of time slots by TDMA to as many transmitters on the same channel which allows only single transmitter to access the channel at any instant of time whereas the spread spectrum CDMA allows all of the transmitters available to access the channel in the same instant of time.

**FDMA: FREQUENCY DIVISION MULTIPLE ACCESS**

FDMA is used in the analog cellular system where the available bandwidth is divided into a number of non-overlapping frequency channels whereas the users assigned the channel upon request. FDMA channels have relatively narrow bandwidth (30 KHz) in which each channel supports only one circuit per carrier. FDMA systems have higher cost because of cell site system as of having the single channel or the single carrier. The subscriber cost becomes high as the duplexers in both the transmitter and receiver side increases. It has costly band pass filter which eliminates the spurious radiation.

**TDMA: TIME DIVISION MULTIPLE ACCESS**

TDMA system is applied only in the digital radio system which uses the single carrier frequency with various users where each of the users uses the non-overlapping time slots. Duplexers are not used in TDMA but the use of adaptive equalization maintains the data transmission rate higher. The user of the TDMA system uses the data transmission in the
discrete burst which maintains the low battery consumption as well as the handoff process will remain simple during the idle time slots.

**CDMA: CODE DIVISION MULTIPLE ACCESS**

This type of multiple access schemes is the special case of spread spectrum multiple access where the narrowband message signal spreads through the PN code sequence. CDMA technology provides the resistance against the interference and multipath fading whereas the channel data rates are higher during its access. The same frequency is being shared by all the active users which may transmit simultaneously and will have independent PN codes. This technique can use either FDD or TDD techniques.

**1.3 WIRELESS CHANNEL**

Wireless transmission system uses free space as its transmission medium. The radio propagation is not as smooth as in wire medium because the received signal is not only coming directly from the transmitter but also coming from the combination of reflected, diffracted and scattered waves with multiple copies. Diffraction occurs when the radio path between the transmitter and receiver is obstructed by a surface that has sharp edges. Scattering occurs when the medium consists of objects with dimensions that are small compared to wavelength and where the number of obstacles per unit volume is large.

There are key model parameters that help to characterize the wireless channel such as path loss, shadowing, delay spread, coherence time, coherence bandwidth, doppler spread etc. which are described briefly as per the requirement.

Path loss means loss in power. If the radio signal propagates in space, the signal power will be decreased with the increase in distance. Free space propagation model treats the region between the transmitter and receiver free of objects that absorb or reflect the radio frequency (RF) energy so that this region behaves as a perfectly uniform medium. This type of model can be used in satellite communication. For the long distance path loss model, the received signal power will have exponential decay with the increase in distance i.e. for indoor and outdoor environment [6, 7].

When the signal passing through the medium creates obstruction then the signal power attenuates haphazardly with the distance. The two locations having different surroundings should have to show the variation in the signal different. If it shows the variation then
such phenomena is known as shadowing. Log normal shadowing formula can be expressed as

$$PL(dB) = PL(d_0) + 10\alpha \cdot \log_{10}\left(\frac{d}{d_0}\right) + X_\Omega$$

(1.3.1)

$d_0$ is the reference distance and $X_\Omega$ is the zero mean Gaussian random variable.

1.3.1 Multi-path Fading [6]

Multipath propagation gives rise to small scale fading. The multipath properties of a given environment are usually characterized by power delay profile. Power delay profile denotes the average power of each multipath. The transmission of signals in the terrestrial environment leads to reflection, diffraction and scattering in the propagation environment. Because of this reflection, diffraction and scattering phenomena, several copies of the original transmit signal are received with various delay, attenuation and phase shift [7].

Fig. 1: Reflected, diffracted, and scattered wave component
1.3.1.1 Power Delay Profile and RMS Delay Spread [8, 9]

The squared absolute value of the channel impulse response which can be obtained by transmitting an impulse over the channel can be expressed as power delay profile [9].

\[ P(t) = \sum_{k=0}^{N-1} a_k^2 \delta(t - \tau_k) \]

(1.3.1.1.1)

Here \( k \) is the indexed path, \( a_k \) is the path gain, \( \tau_k \) is the delay path and \( N \) is the number of path.

The power of the signals received at the receiver is known as power profile. Multiple signal copies arriving over the interval of time and this time period is known as delay spread. Power delay profile helps to obtain the multi-path channel parameters i.e. average delay \((\bar{\tau})\) and RMS delay spread \((\sigma_\tau)\). Average delay and the root mean square delay can be expressed as [8]

\[ \bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k) \tau_k}{\sum_k P(\tau_k)} \]

(1.3.1.1.2)

\[ \sigma_\tau = \sqrt{\bar{\tau}^2 - (\bar{\tau})^2} \]

(1.3.1.1.3)

\[ \frac{\bar{\tau}^2}{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k) \tau_k^2}{\sum_k P(\tau_k)} \]

(1.3.1.1.4)

1.3.1.2 Coherence Bandwidth [9, 10]

The channel that is enjoying flat fading as opposed to frequency selective fading can be called it as coherence bandwidth. During the particular point there will be the significant change in the channel and the response of the channel is downgraded [9].
If the signal bandwidth is smaller than the coherence bandwidth \( B_S < B_C \) then there is no distortion in the output of the channel which means the flat fading distortion. If \( B_S > B_C \) then at the centre all frequency passes but around the edge there will be distortion. When whole signal passing through the channel is without distortion then it is called the frequency selective distortion.

### 1.3.1.3 Coherence Time and Doppler Spread [8]

Time over which the channel is approximately constant is the coherence time which is related to Doppler spread. It is also the measure of time duration over which the channel occurs slow fading. Doppler spread is the change in the frequency of the electromagnetic wave arising due to the relative motion between the transmitter and the receiver.

\[
T_C \approx \frac{1}{f_{D,max}}
\]

(1.3.1.3.1)

Where maximum Doppler spread is given by

\[
f_D = \frac{v \cdot f_c}{c} \times \cos(\theta)
\]

(1.3.1.3.2)

\[
f_{D,max} = \frac{v \cdot f_c}{c} \quad \text{occurs for } \theta = 0, \pi
\]

(1.3.1.3.3)

Where \( v \) is velocity of the mobile terminal, \( c \) is the velocity of light in free space, \( \theta \) is the angle between velocity of mobile and line joining mobile and base station and \( f_c \) is the carrier frequency.
1.3.2 Flat Fading

If the channel bandwidth is greater than the signal bandwidth then such type of signal is referred as flat fading. Flat fading channel brings challenges in gain variation and in frequency spectrum. Deep fade is due to the distortion gain so significant power must have to be increased in some frequencies. If the signal results the destructive interference then it may result the deep nulls in the power spectrum [11].

1.3.3 Frequency Selective Fading

Frequency selective fading occurs if channel bandwidth is smaller than the signal bandwidth. If the overall symbol duration is greater than the RMS delay spread \( \left( \sigma_T \gg T_{sym} \right) \) then ISI occurs. To combat against ISI, Equalization technique is the must which helps to reverse the distortion basically on the receiver side [11].

1.3.4 Fast Fading

If the symbol time is greater than the Coherence time \( (T_s > T_c) \) then fast fading occurs. As the symbol duration is wide in low data rates, it is too much dangerous to fast fading. Due to the movement or because of the time varying nature, channel bandwidth also changes [6, 11].

1.3.5 Slow fading

Slow fading refers to that fading in which there is the insignificant change in Doppler shift due to the shift in frequency if the signal bandwidth is much higher than the Doppler spread. This effect will create the changes in the channel response slowly [11].

1.4 FADING MODELS

1.4.1 Rayleigh Fading Model [12, 13]

It is caused through the multipath components having its constructive part as well as the destructive nature occurring in the flat fading channels which is applicable in between the transmitter and receiver side if there is the condition of no line of sight. This model is most reasonable to apply for the ionosphere and troposphere environment creating for signal propagation and the area where the effect of radio signals being heavily
constructed in the urban areas. The probability density function for the Rayleigh fading can be expressed as

\[ P_a(r) = \frac{\alpha}{\sigma^2} e^{\left(-\frac{\alpha^2}{2\sigma^2}\right)} \quad \text{for} \ 0 \leq \alpha \leq \infty \]  

(1.4.1.1)

\( \sigma^2 \) represents time average power of the received signal.

**1.4.2 Ricean Fading Model [12]**

This is the model same as that of Rayleigh fading except that the Ricean fading consists of the dominant component which is strong enough which can be known with it as a non-fading signal or can be called it as LOS component. Ricean distribution can be represented mathematically as

\[ P(r) = \frac{r}{\sigma^2} e^{-\frac{(r^2+A^2)}{2\sigma^2}} \frac{Ar}{\sigma^2} \quad \text{for} \ A \geq 0; \ r \geq 0 \]  

(1.4.2.1)

Where \( A \) represents the peak amplitude and \( I(\cdot) \) is the Bessel function of the first kind and Zero order.

**1.4.3 Nakagami-m Distribution**

Nakagami distribution or the nakagami-m distribution is a probability distribution related to gamma distribution. It consists of two parameters. One is the shape parameter, \( \mu \) and the other parameter is the controlling speed, \( \omega \).

The nakagami-m distribution having the PDF is in the form of

\[ f(X, \mu, \omega) = \frac{2\mu^\mu}{\tau(\mu)\omega^\mu} X^{2\mu-1} e^{\frac{\mu}{\omega}X^2} \quad \text{for} \ X \geq 0 \]  

(1.4.3.1)

The CDF function is given by

\[ F(X, \mu, \omega) = P\left( \mu, \frac{\mu}{\omega}X^2 \right) \]
P is the incomplete gamma function. In addition to the m-distribution, the following compact form of distribution is presented by Nakagami in 1940 and is named as n-distribution.

\[
P(R) = \frac{2R}{\sigma} e^{\left(\frac{R^2 + R_0^2}{\sigma}\right)} I_0\left(\frac{2RR_0}{\sigma}\right)
\]

(1.4.3.3)

and

\[
P(R) = \frac{2R}{\sigma} e^{\left(\frac{a^2}{\sigma}\right)} I_0\left(\frac{R^2}{2} \left(1 - \frac{1}{\beta a}\right)\right)
\]

(1.4.3.4)

Which is called q-distribution.

1.5 INTRODUCTION TO OFDM

The excessive growth of data, voice and video over the wireless media fuelled an increasing demand for the high data rates. To meet these demands, OFDM is at the centre of attraction because of its attractive features. OFDM is a well-known multi-carrier modulation scheme for the transmission of signals over wireless channels. The main function of the OFDM is to convert the frequency selective fading channel into a collection of parallel flat fading channel. The great advantage to use OFDM is to increase robustness against narrowband interference and the frequency selective fading channel. The sub-carriers are orthogonal to each other in the OFDM system which led these sub-carriers to overlap in frequency with the adjacent carriers without causing interference.

The idea of OFDM system is not totally new. It was first studied and researched in the period between 1950s and 1960s. The patent right for the OFDM system was first obtained by Chang in 1966 [16]. The implementation which consisted of large number of oscillators in the transmitter side and the collection of matched filters in the receiver as the number of sub-carriers which made the implementation complicated and expensive. Later in 1971 Weinstein and Ebert confirmed that OFDM could be analysed and performed by the IDFT and DFT operation [17]. The evolution brought by the Weinstein
and Ebert led the oscillators and filters side-lined and made the researchers to attract the attention in the field of OFDM system. Nowadays, OFDM system is the best standardization communication technique by many wired and wireless applications.

The initial IEEE physical standard for WLANs was testified in June 1997 and specified as one medium access control (MAC) and three physical layers (IEEE 802.11 FHSS, IEEE 802.11 DSSS AND IEEE 802.11 IR). 802.11 DSSS originally supported the data rates of both 1 Mbps and 2 Mbps. Because of the excessive demand for the higher throughput, high data rate DSSS (IEEE 802.11b) in July 1998 was approached for the standardization which increases the data rate up to 11 Mbps. Side by side 802.11a and 802.11b standards were created simultaneously. The physical standard for IEEE 802.11a was chosen for OFDM which is now considered for the 4G system. The analytical and the experimental results of OFDM system over the mobile radio channel was proposed by casas [20, 21]. Different fading channel conditions on the basis of frequency offset has been proposed by different authors. The recent advancement of OFDM system are also presented on the basis of the art of collection of works which has been compiled and edited by S Hara and R Prasad [5], Taha and Salleh [18], Taewon Hwang [19] in the year 2009 as well as Joshi and Saxena [15] in 2013.

1.6 ORGANIZATION OF THESIS

Chapter 1 introduces the background and introduction of the development of wireless networks with its multiple access techniques. The brief descriptions of channel parameters are analysed along with its multipath propagation.

Chapter 2 describes the fundamentals of OFDM system and highlights the importance of orthogonality in multicarrier as well as the use of cyclic prefix. System models of OFDM system along with its mathematical analysis on modulation and demodulation parts are described and analysed. Benefits, problems and the application of OFDM system were highlighted in this chapter.

Chapter 3 presents the literature survey where the introduction of timing synchronization and the effects of STO were analysed mathematically. Different timing offset estimation proposed by different writers was presented mathematically.
Chapter 4 analyse the simulation results of STO estimation based on CP and the training symbol which helps in finding the maximum in case of correlation and minimum in case of difference as well as other timing offset estimation methods to find the timing metric.

Chapter 5 describes the brief conclusions of the chapter and the future scope.
CHAPTER 2

OFDM SYSTEM

2.1 OFDM SYSTEM MODEL [22]

The figure (2) shows the base-band OFDM system model. The operation in which the data stream is first encoded by the channel encoder is called the forward error correction coding.

In order to protect against the bursty errors, the encoded data bits from the FEC are interleaved where the main principle of interleaving operation is to randomize the burst errors in which they can be solved at the receiver side by using a decoding operation. The interleaver that we have used in this block diagram is the block interleaver. The binary sequence of data is applied to serial-to-parallel converter which takes the encoded data bits and put these data bits into $N$ parallel carriers. $N$ is the number of sub-carriers. The data obtained from S/P acts as a modulator which is then applied to bit-to-symbol mapping operation. The modulation defined on the standards giving the different data
rates will be implemented depending on the data rate requirement. OFDM can use the modulation technique as QAM, PSK, QPSK or DQPSK.

The signal values obtained from the OFDM system via IFFT operation is illustrated with mathematical equation. Let \( X_m \) is the data vector having the \( N \) number of QPSK symbols represented as

\[
X_m(k) = (X_m(0), X_m(1) \ldots X_m(N-1))
\]

(2.1.1)

The equivalent complex base band representation in the one OFDM symbol after having the IFFT operation can be expressed as

\[
x_m(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_m(k) e^{j2\pi kn/N}
\]

(2.1.2)

Here \( N \), \( N_{cp} \), \( n \) and \( m \) represents the size of FFT, length of CP, time index in discrete form and the symbol timing index respectively.

![Fig. 3: Addition of CP to the OFDM symbol](image)

After IFFT, there is Cyclic Prefix (guard interval) addition and is parallel to serial block. CP is required to combat ISI and ICI which may arise due to multipath transmission. Figure (3) shows the CP addition. In CP addition, the last of the OFDM symbol is added at the beginning of the symbol. After adding CP, the signal goes for the parallel-to-serial converter which is then ready for the signal transmission.
At the receiver part, after having the serial-to-parallel operation with excluding the CP, an FFT operation having $N$ point is operated in order to get the received symbol data $\hat{X}_m(k)$ which is expressed as

$$\hat{X}_m(k) = \sum_{n=0}^{N-1} r_m(n) e^{-j2\pi kn}/N$$

(2.1.3)

$$r_m(n) = x_m(n) + awgn(n)$$

(2.1.4)

$r_m(n)$ is the received signal.

### 2.2 FUNDAMENTALS OF OFDM

#### 2.2.1 ORTHOGONALITY

Mathematical analysis is done for finding the Orthogonality where the appropriate conditions must have to be considered between the sub-carriers which are presented below. Each sub-carrier located in the FFT interval has exactly an integer number of cycles. In between the adjacent sub-carriers, the total number of cycles differs by one.

$$\int_{t_s}^{t_s+T_s} e^{-j2\pi k/T_s} (t - t_s) \cdot \sum_{n=0}^{N-1} d_n e^{j2\pi n/T_s} (t - t_s) dt$$

(2.2.1.1)

$$= \sum_{n=0}^{N-1} d_n \int_{t_s}^{t_s+T_s} e^{j2\pi n/k} (t - t_s) dt = d_k T_s$$

(2.2.1.2)

Figure (4) shows that subcarriers of an OFDM signal can overlap which will be able to make the complete use of spectrum. The peak presented in each subcarrier spectrum shows that the power present in all the other subcarriers will be remained zero.
2.3 APPLICATIONS OF OFDM SYSTEM

OFDM has been interlinked with the wireless communication standards which have been approved by the groups of IEEE standard i.e. Wi-Fi (IEEE 802.11 standards), WiMAX (IEEE 802.16 standards). In present context, OFDMA is one of the most outstanding, reliable, efficient and demanding radio transmission methods which are applicable for LTE and advanced system such as telecommunication.

2.3.1 IEEE 802.11 Standards [25, 26, 27, 28]

IEEE802.11 standardization group uses OFDM as a wireless local area network as the basis for 5GHz having the date rates of 6Mbps to 54Mbps in 1999. This standard becomes the first one to use OFDM in packet based communication while for the continuous transmission; the use of OFDM is still not in use excessively. Table 1 shows the parameters of IEEE 802.11 standards
Table 1: Parameters of IEEE 802.11 Standards

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation</td>
<td>BPSK, QPSK, 16-QAM, 64-QAM</td>
</tr>
<tr>
<td>Data Rates</td>
<td>6, 9, 12, 18, 24, 36, 36, 46, 54 Mbps</td>
</tr>
<tr>
<td>Total no. of sub-carriers (pilot + data)</td>
<td>4 + 48 = 52</td>
</tr>
<tr>
<td>Coding Rate</td>
<td>1/2, 2/3, and ¾</td>
</tr>
<tr>
<td>Guard Interval</td>
<td>800 ns</td>
</tr>
<tr>
<td>OFDM Symbol Duration</td>
<td>4 μs</td>
</tr>
<tr>
<td>Channel Spacing</td>
<td>20 MHZ</td>
</tr>
<tr>
<td>Sub-carrier Spacing</td>
<td>312.5 KHZ</td>
</tr>
</tbody>
</table>

2.3.2 IEEE 802.16 Standards [29, 30]:

In 2003, IEEE 802.16a is the standard proposed for fixed broadband wireless system in metropolitan area networks (MAN'S). This standard was launched with the improvement which includes PHY and medium access control (MAC). PHY layer was modelled and outlined for no line of sight (NLOS) communication having the frequency band of 2-11GHz. Even though IEEE 802.16a was still applicable for fixed wireless network but the IEEE 802.16 task group is till now working on the radical and extreme change in mobile wireless metropolitan area networks. IEEE 802.16 which was developed and originated helps in creating the uses in MAN were attempting to replace the last mile wired access with the cable modems and digital subscriber line by broad band wireless networks.

Table 2: Parameters of IEEE 802.16- 2004 Standard

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation</td>
<td>BPSK, QPSK, 16-QAM, 64-QAM</td>
</tr>
<tr>
<td>Sampling Frequency (MHZ)</td>
<td>2, 4, 6.32, 8</td>
</tr>
<tr>
<td>Coding Rate</td>
<td>1/2, 2/3, 3/4, 5/6</td>
</tr>
<tr>
<td>Symbol Duration $T_u , \mu$s</td>
<td>128, 64, 40, 32</td>
</tr>
<tr>
<td>Sub-carrier Spacing</td>
<td>7.81, 16.6, 25.0, 31.3</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>1.75, 3.5, 5.5, 7</td>
</tr>
<tr>
<td>FFT Size</td>
<td>256</td>
</tr>
</tbody>
</table>
### Table 3: Parameters of IEEE 802.16e-2005 Standard

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation</td>
<td>BPSK, QPSK, 16-QAM, 64-QAM</td>
</tr>
<tr>
<td>Sampling Frequency (MHz)</td>
<td>1.9, 5.6, 11.2, 22.4</td>
</tr>
<tr>
<td>Coding Rate</td>
<td>1/2, 2/3, 3/4, 5/6</td>
</tr>
<tr>
<td>Symbol Duration $T_s \mu s$</td>
<td>91.4</td>
</tr>
<tr>
<td>Sub-carrier Spacing</td>
<td>10.9</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>1.75, 3.5, 5.5, 7</td>
</tr>
<tr>
<td>FFT Size</td>
<td>128, 512, 1024, 2048</td>
</tr>
<tr>
<td>Duration of Guard Interval</td>
<td>$\frac{T_s}{4}$, $\frac{T_s}{8}$, $\frac{T_s}{16}$, $\frac{T_s}{32}$</td>
</tr>
</tbody>
</table>

### 2.3.3 MULTI-BAND OFDM [31]:

The combination of OFDM modulation with the transmission of data over different frequency band by Frequency Hopping is referred as multi-band OFDM (MBO). The main primary and the foremost concept of multi-band OFDM are to divide the spectrum into different sub-bands where the data is to be transmitted over every band by OFDM. Multi-band OFDM can be applicable in ultra wide band technology which has been the intense research in this diverse field.

### Table 4: Parameters of Multi-band OFDM

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFT Size</td>
<td>128</td>
</tr>
<tr>
<td>Length of CP (nsec)</td>
<td>60.9</td>
</tr>
<tr>
<td>Data Rate (Mbps)</td>
<td>55, 80, 110, 160, 200, 320, and 480</td>
</tr>
<tr>
<td>Length of Guard Interval (nsec)</td>
<td>9.5</td>
</tr>
<tr>
<td>Sub-carrier Spacing (MHz)</td>
<td>3.2</td>
</tr>
<tr>
<td>Symbol Duration (nsec)</td>
<td>312.5</td>
</tr>
<tr>
<td>Bandwidth (MHz)</td>
<td>528</td>
</tr>
</tbody>
</table>
2.3.4 DIGITAL AUDIO and VIDEO BROADCASTING [32, 33]:

Digital Audio Broadcasting which uses the three transmission modes was first standardized by ETSI launched in 1995 uses the techniques of OFDM. OFDM is primarily applied for single carrier as well as in the multicarrier system so that DAB uses the OFDM as a single frequency network which greatly assists in the robustness of spectral efficiency.

Table 5: DAB Standard

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation</td>
<td>DQPSK</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEC</td>
<td>Convolution Code</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of Sub-carriers (N)</td>
<td>1546</td>
<td>768</td>
<td>384</td>
</tr>
<tr>
<td>Length of Guard Interval ($T_g$)</td>
<td>$\frac{T_s}{4}$ (250 ( \mu s ))</td>
<td>$\frac{T_s}{4}$ (62.5 ( \mu s ))</td>
<td>$\frac{T_s}{4}$ (31.25 ( \mu s ))</td>
</tr>
<tr>
<td>Information Trnsmission</td>
<td></td>
<td></td>
<td>2.4 Mbps</td>
</tr>
<tr>
<td>Sub-carrier Separation ($\Delta_f$)</td>
<td>3.968 KHZ</td>
<td>1.984 KHZ</td>
<td>0.492 KHZ</td>
</tr>
<tr>
<td>Useful Symbol length ($T_u$)</td>
<td>1 ms</td>
<td>250 ( \mu s )</td>
<td>125 ( \mu s )</td>
</tr>
<tr>
<td>Bandwidth</td>
<td></td>
<td></td>
<td>1.536 MHZ</td>
</tr>
</tbody>
</table>

Digital Video Broadcasting was first launched in 1993 so as to broadcast the videos in television via satellites, cables etc. but the previously launched standard was modified in 1997 as the terrestrial Digital Video Broadcasting which basically uses the two modes of transmission as 2K and 8K.
Table 6: DVB-T Standard

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Mode 2k</th>
<th>Mode 8k</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation</td>
<td>QPSK</td>
<td>16 and 64-QAM</td>
</tr>
<tr>
<td>FEC</td>
<td>Inner Code- Convolution Code (R=1/2, 2/3, 3/4, 5/6, 7/8)</td>
<td>Outer Code- Reed Solomon Code (204, 168)</td>
</tr>
<tr>
<td>No. of Sub-carriers (N)</td>
<td>1705</td>
<td>6817</td>
</tr>
<tr>
<td>Length of Guard Interval ((T_g))</td>
<td>(\frac{T_u}{16} (14 \mu s), \frac{T_u}{32} (7 \mu s))</td>
<td>(\frac{T_u}{16} (56 \mu s), \frac{T_u}{32} (28 \mu s))</td>
</tr>
<tr>
<td>Information Transmission</td>
<td>4.98- 31.67 Mbps</td>
<td></td>
</tr>
<tr>
<td>Sub-carrier Separation ((\Delta f))</td>
<td>4.464 KHZ</td>
<td>1.116 KHZ</td>
</tr>
<tr>
<td>Useful Symbol length ((T_u))</td>
<td>224 \mu s</td>
<td>896 \mu s</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>7.61 MHZ</td>
<td></td>
</tr>
</tbody>
</table>

2.4 BENEFITS OF OFDM SYSTEM [5, 2]:

- **Use of efficient bandwidth:**
  Orthogonality is maintained in between the sub-carriers which help to overlap the carriers.

- **Robustness when combating with the narrow band co-channel interference:**
  because in single carrier system the entire system will fail if fading occurs but in multi-carrier a small portion of the sub-carriers will be affected.

- **Immunity to delay spread:**
  The existence of some of the components of any types of symbol in the domain of the next symbol which occur distortion resulting to the inter-symbol interference so called as delay spread. Differences in path lengths of multi path channel depend on delay spread. The guard interval has to be introduced whose time duration is longer than the duration of delay spread so that ISI can be removed.

- Almost each sub-channel is flat fading which needs a single tap equalizer to eradicate the effect of channel [34].

- **Resistance to frequency selective fading due to parallel transmission** [1].
2.5 PROBLEMS ASSOCIATED WITH OFDM:

OFDM is the growing demand among the latest technology which has always been the area of interest. It’s been more concern for the better quality. Besides great achievement, there are some certain issues which have always been the scope of study.

- **TIMING SYNCHRONIZATION [34]:**
  The ultimate task of finding the precise moment of the individual OFDM symbols start and the end is the Symbol Synchronization. It refers to the timing errors and carrier frequency offset which is highly sensitive in OFDM system. High sensitivity is due to the use of IFFT and FFT for modulation and demodulation at transmitter and receiver side as per their respective point. To recover these errors various algorithms have been proposed which are discussed in the next chapter.

- **FREQUENCY SYNCHRONIZATION [35]:**
  It refers to the sampling frequency synchronization and carrier frequency synchronization. Carrier frequency offset arises between the sub-carriers if the orthogonality between the sub-carriers will be destroyed.

- **HIGH PEAK TO AVERAGE POWER RATIO [36]:**
  Efficiency will be reduced by high PAPR which degrades the total performance of the system.

2.6 PROBLEM FORMULATION

There were many numerous applications which is being successfully implemented in Orthogonal frequency division multiplexing but the core element in OFDM system in any wireless transmission scheme is the synchronization at the receiver where the orthogonality of the sub-carriers will be disturbed because of frequency and timing synchronization. The principle drawbacks of OFDM system is the Synchronization which leads to the introduction of ISI and ICI so that there will be the loss of orthogonality between the sub-carriers and can void the advantages of OFDM system. The intense research in this field made the researchers to give the best performance in timing offset estimation. In spite of its major challenges, it’s one of the most interesting and the relevant field in today’s LTE and 4G technology. Timing offset estimation is illustrated in the next chapter in an elaborated form.
2.7 SUMMARY

Chapter 2 describes about the system model of OFDM system with its modulation and demodulation part presenting the mathematical analysis. The basic fundamentals of OFDM system with its application, benefits and the problems were summarized to enhance the timing offset estimation. Frequency selective fading were lessened by the parallel transmission of data streams which permits the data rates with higher speed producing high spectral bandwidth. This chapter also introduces the different standards used in OFDM system.

The upcoming chapter introduces the timing offset estimation in OFDM system with its effects. These topics were analysed and illustrated with mathematical expressions.
CHAPTER 3

LITERATURE SURVEY

3.1 TIMING SYNCHRONIZATION IN OFDM SYSTEM

Synchronization acts as a functional element in the design of digital communication system. Timing synchronization in single carrier system refers to frame or symbol whereas timing synchronization in multicarrier refers to find the correct initial point of FFT window at the receiver side. The major issues in the OFDM system are the timing synchronization. The imperfect synchronization destroys the orthogonality of sub carriers and leads to the poor performance in the system. The synchronization in OFDM system can be realized based on data aided and non-data aided methods. If we use the data aided method, training sequence and pilot symbols were supplied so as to achieve the rapid synchronization. The training signals can be transmitted continuous or periodically. The training signals can be identified through its shape, autocorrelation function spectrum and the related parameters. This type of method is mostly applied for the public access system. The system which uses the packet data transmission as well as the local area networks can achieve the synchronization with low complexity and high accuracy.

The method having non-data aided is basically known as the blind synchronization where the estimation is performed based on the use of cyclic prefix. The large number of samples must have to be achieved to get the appropriate estimation in which the length of the cyclic prefix and the signal to noise ratio value will affect the performance of the estimation. The system performance can be poor if the ISI channel is obtaining the interference from the previous symbol in the cyclic prefix. Data aided methods mainly concentrate on the wide area research in modem WLAN system. We include the effects of timing offset, need of timing synchronization and its algorithm, fine timing, coarse timing along with the reviews based on timing synchronization which has been proposed by different authors.

3.2 SYMBOL TIMING OFFSET EFFECTS [37]

Symbol timing estimation in OFDM system helps to find the starting point of the FFT window at the receiver side. The effects of STO are determined depending on the location
of the estimated starting point of OFDM symbol. There are four different cases of timing offset which is illustrated below.

\[ T_g \quad T_{ul} \]

\begin{align*}
\text{CP} & \quad i^{th} \text{ OFDM Symbol} & \quad \text{CP} & \quad (i + 1)^{th} \text{ OFDM Symbol} \\
\tau_{max} & \quad \text{Case I} & & \\
\quad & \quad \text{Case II} & \\
\quad & \quad \text{Case III} & \\
\quad & \quad \text{Case IV} & 
\end{align*}

Fig. 5: STO effects in different cases

**Case I:** consider the case when there is no timing error. First case refers to that when the estimated starting point of the OFDM symbol coincides with the exact timing preserving the orthogonality among the subcarriers so that the OFDM symbol can exactly be recovered without having any types of interference. One complete OFDM symbol during the time after removing the cyclic prefix, Received signal can be expressed as;

\[ r_i = \{r(i, 0), (i, 1), \ldots, (i, N - 1)\} \]

(3.2.1)

\( r_i \) symbolizes the current vector of the FFT operation. \( r(i, n) \) is the \( n^{th} \) sample of \( i^{th} \) OFDM symbol which can be represented mathematically as;

\[ r(i, n) = \sum_{l=0}^{L-1} h_l(\tau)x(i, n - \tau_l) + \omega(i, n) \]
\[ f o r \ n = -G, -G + 1, ..., 0, 1, ..., N - 1 \] (3.2.2)

\( x(i, n) \) is the transmitted OFDM symbol, \( h_i(n) \) is the impulse response in multi path fading and \( w(i, n) \) is the Zero mean. The output of the FFT can be express as:

\[ y(i, p) = \sum_{n=0}^{N-1} r(i, n)e^{(-j2\pi p n/N)} \ for \ p = 0, 1, ..., N - 1 \] (3.2.3)

Where \( r(i, n) \) is substituted then the equation becomes

\[ y(i, p) = \sum_{n=0}^{N-1} \left\{ \sum_{\tau=0}^{L-1} h_i(\tau)x(i, n - \tau) + w(i, n) \right\} e^{(-j2\pi p n/N)} \ for \ p = 0, 1, ..., N - 1 \] (3.2.4)

Separating the AWGN terms, the above equation can be written as

\[ y(i, p) = \sum_{n=0}^{N-1} \sum_{\tau=0}^{L-1} h_i(\tau)x(i, n - \tau)e^{(-j2\pi p n/N)} + \sum_{n=0}^{N-1} w(i, n)e^{(-j2\pi p n/N)} \]

\[ f o r \ p = 0, 1, ..., N - 1 \] (3.2.5)

Putting the values of \( x(i, n - \tau) \), we get

\[ y(i, p) = \sum_{n=0}^{N-1} \sum_{\tau=0}^{L-1} h_i(\tau) \left\{ \frac{1}{N} \sum_{k=0}^{N-1} X(i, k)e^{j2\pi k(n-\tau)N} \right\} e^{(-j2\pi p n/N)} + w(i, p) \]

\[ f o r \ p = 0, 1, ..., N - 1 \] (3.2.6)
By interchanging the summation

\[
y(i, p) = \left( \frac{1}{N} \right) \sum_{k=0}^{N-1} X(i, k) \left\{ \sum_{l=0}^{L-1} h_l(\tau) e^{-\frac{(j2\pi k \tau)}{N}} \right\} \left\{ \sum_{n=0}^{N-1} 1 \times e^{\frac{j2\pi n(k-p)}{N}} \right\} + w(i, p)
\]

for \( p = 0, 1, \ldots, N - 1 \)  

(3.2.7)

\[
H(k) = \sum_{l=0}^{L-1} h_l(\tau) e^{-\frac{(j2\pi k \tau)}{N}}
\]

Substituting the equation \( H(k) \):

\[
y(i, p) = \left( \frac{1}{N} \right) \sum_{k=0}^{N-1} X(i, k) H(k) \sum_{n=0}^{N-1} 1 \times e^{\frac{j2\pi n(k-p)}{N}} + w(i, p)
\]

for \( p = 0, 1, \ldots, N - 1 \)  

(3.2.8)

When rearranging the above equation

\[
y(i, p) = X(i, p)H(p) + \left( \frac{1}{N} \right) \sum_{k=0, k \neq p}^{N-1} X(i, k) H(k) \sum_{n=0}^{N-1} 1 \times e^{\frac{j2\pi n(k-p)}{N}} + w(i, p)
\]

for \( p = 0, 1, \ldots, N - 1 \)  

(3.2.9)

The complete output of the FFT of the final result can be expressed as

\[
y(i, p) = X(i, p)H(p) + w(i, p) \quad \text{for } p = 0, 1, \ldots, N - 1
\]

(3.2.10)

With the help of the following property
\[
\sum_{n=0}^{N-1} e^{j2\pi n(k-p)/N} = e^{j2\pi n(k-p)N^{-1}} \sin(\pi(k - p)) \\
= \begin{cases} 
N, & \text{for } k = p \\
0, & \text{for } k \neq p 
\end{cases}
\]

(3.2.11)

If there is not the presence of timing and frequency offset then distortion will be free. The presence of timing offset will not led the samples in the FFT window to start from the exact point.

**Case II:** The next case is when the estimated starting point of the OFDM symbol is before the exact point, yet after the end of the channel response to the previous OFDM symbol. Let us consider the received signal in the frequency domain by taking the FFT on the time domain to see the effects of symbol timing offset having the received samples \( r(i, n + \delta) \). Now the equation can be represented as

\[
Y(i, p) = \sum_{n=0}^{N-1} r(i, n + \delta)e^{-j2\pi pn/N}, \quad \text{for } p = 0, 1, ..., N - 1
\]

(3.2.12)

Putting the values of \( r(i, n + \delta) \);

\[
Y(i, p) = \sum_{n=0}^{N-1} \left( \sum_{l=0}^{L-1} h_l(\tau)x(i, n + \delta - \tau_l) + w(i, n + \delta) \right) e^{-j2\pi pn/N}
\]

\[\quad \text{for } p = 0, 1, ..., N - 1
\]

(3.2.13)

While separating the AWGN terms

\[
Y(i, p) = \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} h_l(\tau)x(i, n + \delta - \tau_l)e^{-j2\pi pn/N} + \sum_{n=0}^{N-1} w(i, n + \delta)e^{-j2\pi pn/N}
\]

\[\quad \text{for } p = 0, 1, ..., N - 1
\]

(3.2.14)
Extracting the values of \(x(i, n + \delta - \tau_i)\) and interchanging of summation

\[
Y(i, p) = \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} h_l(\tau) \left\{ \frac{1}{N} \sum_{k=0}^{N-1} X(i, k) e^{j2\pi k(n+\delta-\tau_i)/N} \right\} e^{-j2\pi pn/N} + w(i, p)
\]

for \(p = 0, 1, ..., N - 1 \)

(3.2.15)

\[
Y(i, p) = \frac{1}{N} \sum_{k=0}^{N-1} X(i, k) e^{j2\pi k\delta/N} \left\{ \sum_{l=0}^{L-1} h_l(\tau) e^{-j2\pi kl/N} \right\} \left\{ \sum_{n=0}^{N-1} 1 \times e^{j2\pi n(k-p)/N} \right\} + w(i, p)
\]

for \(p = 0, 1, ..., N - 1 \)

(3.2.16)

Substituting \(H(k)\)

\[
Y(i, p) = \frac{1}{N} \sum_{k=0}^{N-1} X(i, k) e^{j2\pi k\delta/N} H(k) \left\{ \sum_{n=0}^{N-1} 1 \times e^{j2\pi n(k-p)/N} \right\} + w(i, p)
\]

for \(p = 0, 1, ..., N - 1 \)

(3.2.17)

Breaking the summation parts in two terms

\[
Y(i, p) = e^{j2\pi p\delta/N} X(i, p)H(p) + \frac{1}{N} \sum_{k=0, k \neq p}^{N-1} X(i, k) e^{j2\pi k\delta/N} H(k) \left\{ \sum_{n=0}^{N-1} 1 \times e^{j2\pi n(k-p)/N} \right\} + w(i, p)
\]

for \(p = 0, 1, ..., N - 1 \)

(3.2.18)

Final result becomes

\[
Y(i, p) = e^{j2\pi p\delta/N} X(i, p)H(p) + w(i, p) \text{ for } p = 0, 1, ..., N - 1
\]

(3.2.19)
From this above final result the orthogonality among the subcarriers will be preserved but there exist the a phase offset proportional to the symbol timing offset which can be compensated by a single tap frequency domain equalizer.

**Case III:** This one is the Case prior to the end of the (lagged) channel response to the previous OFDM symbol when the starting point of the OFDM symbol is estimated to exist. This will allow the symbol timing avoid the ISI too early. ISI will led the orthogonality among the sub-carriers destroyed from the previous symbol which will led them to occur ICI.

**Case IV:** The fourth case is that when the starting point of the OFDM symbol is estimated just after the exact point. The current OFDM symbol $x(i,n)$ and the next symbol $x(i+1,n)$ samples are taken for the current FFT operation. The received sampled signal can be expressed as

$$r_i = \{ r(i, \delta), r(i, \delta + 1), \ldots, r(i, N - 1), r(i + 1, -G), r(i + 1, -G + 1), \ldots, r(i + 1, -G + \delta - 1) \}$$

(3.2.20)

$r_i$ can also be written as

$$r_i = \begin{cases} 
  r(i, n + \delta) & \text{for } 0 \leq n \leq N - 1 - \delta \\
  r(i + 1, n - G - N + \delta) & \text{for } N - \delta \leq n \leq N - 1
\end{cases}$$

(3.2.21)

$r_i$ is the current vector for the FFT operation. The output for the FFT operation can be represented as

$$Y(i, p) = \sum_{n=0}^{N-1-\delta} r(i, n + \delta) e^{-j2\pi pn/N} + \sum_{n=N-\delta}^{N-1} r(i + 1, n - G - N + \delta) e^{-j2\pi pn/N}$$

for $p = 0, 1, \ldots, N - 1$

(3.2.22)

Simplify after putting the values of $r(i, n + \delta)$ and $r(i + 1, n - G - N + \delta)$
\[ Y(i, p) = \sum_{n=0}^{N-1-\delta} \sum_{l=0}^{L-1} h_l(\tau) x(i, n + \delta - \tau_l) e^{-j2\pi pn/N} \]
\[ + \sum_{n=0}^{N-1-\delta} w(i, n + \delta) e^{-j2\pi pn/N} \]
\[ + \sum_{n=N-\delta}^{N-1-\delta} \sum_{l=0}^{L-1} h_l(\tau) x(i + 1, n - G - N + \delta - \tau_l) e^{-j2\pi pn/N} \]
\[ + \sum_{n=N-\delta}^{N-1} w(i + 1, n - G - N + \delta) e^{-j2\pi pn/N} \]

(3.2.23)

Now putting the values of \( x(i + 1, n - G - N + \delta - \tau_l) \) and \( x(i, n + \delta - \tau_l) \); we get

\[ Y(i, p) = \sum_{n=0}^{N-1-\delta} \sum_{l=0}^{L-1} h_l(\tau) \left\{ \frac{1}{N} \sum_{k=0}^{N-1} X(i, k) e^{j2\pi k(n+\delta-\tau_l)/N} \right\} e^{-j2\pi pn/N} \]
\[ + \sum_{n=N-\delta}^{N-1-\delta} \sum_{l=0}^{L-1} h_l(\tau) \left\{ \frac{1}{N} \sum_{k=0}^{N-1} X(i + 1, k) e^{j2\pi k(n-G-N+\delta-\tau_l)/N} \right\} e^{-j2\pi pn/N} \]
\[ + w(i, p) + w(i + 1, p) \quad \text{for } p = 0, 1, \ldots, N - 1 \]

(3.2.24)

\[ Y(i, p) = \frac{1}{N} \sum_{k=0}^{N-1} X(i, k) e^{j2\pi k\delta/N} \left\{ \sum_{l=0}^{L-1} h_l(\tau) e^{-j2\pi k\tau_l/N} \right\} \sum_{n=0}^{N-1-\delta} 1 \times e^{j2\pi p n / N} \]
\[ + \left( \frac{1}{N} \sum_{k=0}^{N-1} X(i + 1, k) e^{j2\pi k(\delta-G-N)/N} \left\{ \sum_{l=0}^{L-1} h_l(\tau) e^{-j2\pi k\tau_l/N} \right\} \sum_{n=N-\delta}^{N-1} 1 \times e^{j2\pi p (n-p) / N} \right) + w(i, p) + w(i + 1, p) \]

\[ \text{for } p = 0, 1, \ldots, N - 1 \]

(3.2.25)
Making it simplified we get the result

\[
Y(i, p) = \frac{1}{N} \sum_{k=0}^{N-1} X(i, k) e^{\frac{j2\pi k\delta}{N}} H(i, k) \left\{ \sum_{n=0}^{N-1-\delta} 1 \times e^{\frac{j2\pi n(k-p)}{N}} \right\}
\]

\[
+ \frac{1}{N} \sum_{k=0}^{N-1} X(i + 1, k) e^{\frac{j2\pi k(\delta-G-N)}{N}} H(i + 1, k) \left\{ \sum_{n=N-\delta}^{N-1} 1 \times e^{\frac{j2\pi n(k-p)}{N}} \right\}
\]

\[
+ w(i, p) + w(i + 1, p) \quad \text{for } p = 0, 1, \ldots, N - 1
\]

(3.2.26)

Re-arranging the above equation we get;

\[
Y(i, p) = \frac{N - \delta}{N} X(i, p) H(i, p) e^{\frac{j2\pi p\delta}{N}}
\]

\[
+ \frac{1}{N} \sum_{k=0, k \neq p}^{N-1} X(i, k) H(i, k) e^{\frac{j2\pi k\delta}{N}} \left\{ \sum_{n=0}^{N-1-\delta} 1 \times e^{\frac{j2\pi n(k-p)}{N}} \right\}
\]

\[
+ \frac{1}{N} \sum_{k=0}^{N-1} X(i + 1, k) H(i + 1, k) e^{\frac{j2\pi k(n+\delta-G-N)}{N}} \left\{ \sum_{n=N-\delta}^{N-1} 1 \times e^{\frac{-j2\pi n}{N}} \right\}
\]

\[
+ w'(i, p) \quad \text{for } p = 0, 1, \ldots, N - 1
\]

(3.2.27)

Here \(w'(i, p) = w(i, p) + w(i + 1, p)\)

Applying the identity equation here we could be able to get the following result.

\[
\sum_{n=0}^{N-1-\delta} e^{\frac{j2\pi n(p-k)}{N}} = e^{j2\pi(p-k)} e^{\frac{N-1-\delta}{N}} \frac{\sin\left(\frac{(N - \delta)\pi(k - p)}{N}\right)}{\sin\left(\frac{\pi(k - p)}{N}\right)}
\]

\[
= \begin{cases} 
N - \delta, & \text{for } k = p \\
\text{non zero}, & \text{for } k \neq p 
\end{cases}
\]

(3.2.28)
3.3 TIMING OFFSET ESTIMATION METHODS

There are the different timing offset estimation methods which are based on the training symbol transmission. These are generally called it as a data added methods. Before the transmission of actual information data symbols, preambles are actually transmitted for timing synchronization in the case of data added methods. The timing estimation algorithm helps to find the maxima of auto-correlation of the incoming signal which helps to detect the starting point of the training symbol. There are many more methods related to the timing offset estimation. Some of the timing offset estimation methods are described below on the basis of their training symbol patterns and the timing metric. We basically describe about the fine timing estimation.

3.3.1 Fine Timing

3.3.1.1 Schmidl and Cox Method

Timing synchronization in Schmidl and Cox method [38] is obtained by using a training sequence where the time domain of the first half is equal to its second half. The basic principles in Schmidl and Cox is that as long as the timing estimate is in the CP, the symbol timing errors will have some effects within the signal. Training sequences are to be made identical between the two halves which are to be transmitted by PN sequence on the even frequencies and the zeros which are to be sent on the odd frequencies. The main benefits of using the zeros on the odd frequency is that it helps to distinguish the symbols from the data which could have the values in the odd part so that it plays the effective role in continuous broadcasting system. The next method of obtaining the training symbol having two identical halves is of using the PN sequence of simply the half symbol length. These training symbols should have to be generated by taking the FFT operation and repeat the operation as per its requirement.

Schmidl and Cox proposed the preamble in the form of

$$preamble_{s&c} = \begin{bmatrix} A_{N/2} & A_{N/2} \\ \end{bmatrix}$$

(3.3.1.1.1)

$A_{N/2}$ denotes the length of $N/2$. 

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Let us consider \( N \) be the number of complex samples. The algorithms developed by Schmidl and Cox basically consists of three steps which are followed by three different equations.

\[
P(n) = \sum_{k=0}^{\frac{N-1}{2}} \left( r(n + k)^*r\left(n + k + \frac{N}{2}\right) \right)
\]

(3.3.1.2)

\[
R(n) = \sum_{k=0}^{\frac{N-1}{2}} \left| n + k + \frac{N}{2} \right|^2
\]

(3.3.1.3)

\[
M(n) = \frac{|P(n)|^2}{\left(R(n)\right)^{\frac{3}{2}}}
\]

(3.3.1.4)

Windows length of \( N \) is the number of subcarriers. The value of \( n \) is the starting point which increases. \( P(n) \) is the cross-correlation between two halves and \( R(n) \) is the auto-correlation on the second half time domain. The values of \( P(n) \) and \( R(n) \) should have to be same if the initial point of the window reaches the start of the training symbol along with the CP.

### 3.3.1.2 Minn and Bhargava Method

In order to reduce the uncertainty to evaluate the correct timing Minn and Bhargava [39] proposed the another method which has the training symbol in the form of

\[
preamble_{Minn} = \begin{bmatrix}
A_{\frac{N}{4}} & A_{\frac{N}{4}} & -A_{\frac{N}{4}} & -A_{\frac{N}{4}}
\end{bmatrix}
\]

(3.3.1.2.1)

\( A_{N/4} \) represents \( N/4 \) point IFFT of a modulated PN sequence.

Cross-correlation sequence having the positive and the negative valued parts of the training symbol is stated as
\[ P_2(n) = \sum_{k=0}^{1} \sum_{m=0}^{L-1} r^*(n + 2Lk + m)r(n + 2Lk + m + L) \]

(3.3.1.2.2)

And the auto-correlation sequence of the positive and the negative valued parts of the training symbol is

\[ R_2(n) = \sum_{k=0}^{1} \sum_{m=0}^{L-1} |r(n + 2Lk + m + L)|^2 \]

(3.3.1.2.3)

L is the length of \( A_{N/4} \). So timing metric is finally given as

\[ M_2(n) = \frac{|P(n)|^2}{(R(n))^2} \]

(3.3.1.2.4)

Equation \( R_2(n) \) calculates the half symbol energy having \( N/2 \) samples so that this equation can be replaced by

\[ R_{2,1}(n) = \frac{1}{2} \sum_{m=0}^{N-1} |r(n + m)|^2 \]

(3.3.1.2.5)

We use the equation \( R_{2,1}(n) \) in simulation. The Minn’s method has a fine timing metric which has its peak better than that of Schmidl and Cox at the correct starting point of the OFDM symbol. Some of the samples of correlation execute the negative values which removes the peak plateau of the timing metric which helps to compute the smaller Mean Square Error.

3.3.1.3 Byungjoon Park Method

This is simply the reverse auto-correlation method [40]. In order to avoid the ambiguity which has been occurred in Schmidl and Cox as well as in the minn’s timing offset
estimation, parks proposed the new method which has the training sequence in the form of

\[
preamble_{park} = \begin{bmatrix} A_{N/4} & B_{N/4} & A^*_{N/4} & B^*_{N/4} \end{bmatrix}
\]

(3.3.1.3.1)

\(A_{N/4}\) denotes the sample length of \(A_{N/4}\) having the PN sequence generated by IFFT. \(A^*_{N/4}\) is the transpose of \(A_{N/4}\). \(B_{N/4}\) is designed to be symmetric with \(A_{N/4}\) to get the timing metric having the impulse response.

\[
P(n) = \sum_{k=0}^{N/2} r(n - k) \cdot r(n + k)
\]

(3.3.1.3.2)

\[
R(n) = \sum_{k=0}^{N/2} |r(n + k)|^2
\]

(3.3.1.3.3)

\[
M(n) = \frac{|P(n)|^2}{(R(n))^2}
\]

(3.3.1.3.4)

An even sharper timing metric is produced from park than from schmidl and minn’s but the timing metric produced by park has its two large side lobes which will affect the timing performance. Between two adjacent values \(N/2\) different pairs of product are designed by parks so that the timing metric has its peak value at the correct symbol timing where the values at all other positions remains almost zero.

3.3.1.4 Adebeng B. Awoseyila

Restricted cross-correlation method is the multistage method proposed by Adebeng B. Awoseyila [41, 42]. To achieve the enhanced estimation performance, a simple auto-correlation technique and novel restricted cross-correlation techniques are combined. The
preamble used in the restricted cross-correlation method uses the two identical parts in time domain same as in Schmidl and Cox method which provides the low complexity coarse timing and a wide range integer frequency estimation which could not be obtained by using the training symbols of Minn’s and Park. An auto-correlation method which is same as that of Schmidl and Cox with its integration is taken in the first stage for the coarse timing estimation. Restricted cross-correlation is carried out in the second stage for the fine timing estimation. The equation for the coarse timing estimation is given by;

$$P(n) = \sum_{k=0}^{N/2-1} r^* (n + k) \cdot r \left(n + k + \frac{N}{2}\right)$$

(3.3.1.4.1)

$$M_c(n) = \frac{1}{G + 1} \sum_{k=0}^{6} |P_{s\&c}(n - k) |^2$$

(3.3.1.4.2)

$$\bar{n}_c = \text{argmax}[M_c(n)]$$

(3.3.1.4.3)

$M_c(n)$ is the coarse timing metric which has to perform the operation of the integration of auto-correlation function $P_{s\&c}(n)$ over the length of CP which helps to remove the plateau remaining uncertainty.

$$P_x(n) = \sum_{k=0}^{N-1} r_{corr} (n + k) s^*(k)$$

(3.3.1.4.4)

$$M_{opt}(n) = |P_x(n)|^2 M_c(n)$$

(3.3.1.4.5)

$$\bar{n}_{opt} = \text{argmax}\{M_{opt}(n)\} \quad \text{for } n \in \left\{ \bar{n}_c - \frac{N}{2}, \bar{n}_c + \frac{N}{2} \right\}$$

(3.3.1.4.6)
The total corrected frequency signal is the $r_{corr}$. $T_{th}$ is the threshold to identify the ideal timing, could appropriately chose for all the timing points which are relevant for the ideal timing points to be tracked. $M_{opt}(n)$ is the timing metric. $n_{FFT}$ is the arriving path where all the channel paths were expected to receive the samples within $\lambda + 1$.

3.3.1.5 Seung’s Timing Offset Estimation

Since the park’s method always assumed to be first arrival channel path dominant, the performance will be degraded when this method is compared in the case of fast varying channel. Fast varying channel will not always create the first channel path dominant. In order to give the proper solution to these problem seung [43] proposed a new method having the preamble pattern as

\[
preamble_{seung} = [A \quad B^*]
\]

Here $A$ denotes the preamble sequence having the length $L = N/2$ which will be operated by the Inverse Fast Fourier Transform (IFFT) of Constant Amplitude Zero Auto-Correlation (CAZAC) sequence which has been modulated by QPSK. $B^*$ is the time reversed version of $A$ which shows the complex conjugate of $B$. Instead of using the conventional cyclic prefix, Seung’s method uses the Zero padding for the guard interval.

\[
P(d) = \sum_{k=0}^{N/2-1} r_{d-k} \cdot r_{d-k-1}
\]
Where \( d \) is the timing index to the first sample having the \( 2L \) samples in the sliding window. The timing metric obtained from the seung’s has the peak impulse response at the correct symbol timing point. As in the case of fast varying channel, the timing metric obtained as in the form of impulse has the predefined values which are threshold and are combined in the moving summation block. The point is estimated to be the correct timing point if it contains the maximum value in the moving summation block. The threshold based window method can be used for estimating the interval between peak point and the estimated correct timing point so that the more accurate estimation is obtained having the peak point compensated.

### 3.3.1.6 Double Autocorrelation Method

This method \([44]\) helps to calculate the two correlations which initiate to detect the start of the frame. The mathematical expressions for this method are

\[
P_1(d) = \sum_{n=0}^{N_t-1} r^*(d + n) \cdot r(d + n + N_t)
\]

(3.3.1.6.1)

\[
P_2(d) = \sum_{n=0}^{N_t-1} r^*(d + n) \cdot r(d + n + 2N_t)
\]

(3.3.1.6.2)

Here \( N_t \) represents the length of short symbol. Now its energy can be calculated as

\[
R(d) = \sum_{n=0}^{N_t-1} |r^*(d + n)|^2
\]

(3.3.1.6.3)
The first and the second timing metric can be calculated as

\[ M_1(d) = \frac{(P_1(d))^2}{(R(d))^2} \]

(3.3.1.6.4)

\[ M_2(d) = \frac{(P_2(d))^2}{(R(d))^2} \]

(3.3.1.6.5)

Now the timing metric can be calculated as

\[ M = \text{argmax}(M_1 - M_2) \]

(3.3.1.6.6)

Double Autocorrelation method helps in the detection of the start of the 9th short symbol in the case of ideal condition. The subtraction of the two matrices \( M_1 \) and \( M_2 \) will give the result of the maximum value while starting of 9th short symbol.

### 3.3.1.7 GIB’s Method

This method [45] helps in finding both the timing and frequency offset estimation which basically uses the guard interval. In preamble based method, the main idea is to detect a peak in terms of correlation which focuses some of the methods of timing synchronization where Moving Average (MA) is one of the methods to find timing offset. The mathematical expression can be expressed as

\[ P(d) = \sum_{n=d-L+1}^{d} Re\{c^*(n) \cdot c(n - N)\} \]

(3.3.1.7.1)

d is the timing index in the related timing instant having c as the quantization if received signal. L indicates the length of cyclic prefix and N be the length of the OFDM symbol. The next method is the Exponentially Moving Average which can be formulated with the required equation as
\[ P(d) = \sum_{n=0}^{d} w^{d-k} \text{Re}\{c^*(n) \cdot c(n - N)\} \]

(3.3.1.7.2)

Where \( w \) represents the weighted factor.

### 3.3.1.8 Yang Bo’s Algorithm

After knowing the demerits of all the above algorithms, a Chinese scientist Yang Bo [46] proposed a new method in the year 2009 where the training symbols were designed on the basis of Schmidl and Cox method. The designed preamble can be stated as

\[
\text{preamble}_{yang} = \begin{bmatrix} A_N & B_N^* \end{bmatrix}
\]

(3.3.1.8.1)

The assumption of this algorithm is that \( A^* \) is the conjugate of \( A_{N/2} \) and \( B_{N/2}^* \) is symmetric with \( A^* \). As obtained the plateau in Schmidl and Cox which is one of the demerits can be eliminated by using the Yang Bo’s algorithm. The correlation timing metric for this algorithm can be written as

\[
P_{yang}(n) = \sum_{k=0}^{N/2-1} r(n + k) \cdot r(N + n - k - 1)
\]

(3.3.1.8.2)

The timing metric for this algorithm can be expressed as

\[
M_{yang}(n) = \left( \frac{|P_{yang}(n)|^2}{\left( R_{yang}(n) \right)^2} \right)
\]

(3.3.1.8.3)

The mathematical equation for the \( R_{yang}(n) \) can be written as
Based on this algorithm sharp peaks are obtained which suppresses the side peaks in a lower amount.

### 3.3.1.9 Classen Approach

Classen [47] proposed the two timing synchronization techniques for coarse and fine timing in their respective procedure. Frame synchronization will be obtained through channel impulse response (CIR) which can estimate fine timing. While estimating coarse timing, time and frequency offset were totally unknown and the start of the symbol could be found out through the approach of the periodic structure. This was accomplished by minimizing the metric $\hat{\delta}$ defined by the equation as

$$
\hat{\delta} = \left\{ \sum_{i=\delta}^{N_{g}-1+\delta} (|r_i[n+i]| - |r_i^*[n+N+i]|)^2 \right\}
$$

(3.3.1.9.1)

For finding out the fine timing estimation, the assumptions for CFO and its correction has been made. This technique could be used for the information of CIR to obtain the timing offset correction. The technique was to consider the energy maximization of CIR within the interpolation window. This technique causes the energy of the channel estimate to get maximized. Since interpolation adds no additional information, timing metric was evaluated at the pilot position.

### 3.3.1.10 Shi and Serpedin Algorithm:

Based on the maximum likelihood criteria, Shi and E. Serpedin [48] proposed the new algorithm modifying the algorithm of Minn’s method in 2004 which uses the preamble as follows.

$$
Preamble_{\text{shi\&serpedin}} = \begin{bmatrix}
A_N & A_N & -A_N & A_N \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4}
\end{bmatrix}
$$
represents the pseudo random noise sequence having the length of \( \frac{N}{4} \) where the received sample manipulates four vectors \( r_i \) for \( i = 0, 1, 2, 3 \). The timing metric for the following algorithm can be expressed as

\[
M_{\text{shi\&sher}}(n) = \frac{P_{\text{shi\&sher}}(n)}{\frac{1}{2} \sum_{i=1}^{4} |r_i|^2}
\]

Now the collection of four vectors can be represented in terms of equation.

\[
P_{\text{shi\&sher}}(n) = |R_1(n)| + |R_2(n)| + |R_3(n)|
\]

\[
R_1(n) = r_0(n)^H r_1(n) - r_1(n)^H r_2(n) - r_2(n)^H r_3(n)
\]

\[
R_2(n) = r_1(n)^H r_3(n) - r_0(n)^H r_2(n)
\]

\[
R_3(n) = r_0(n)^H r_3(n)
\]

Transpose conjugate is shown by the notation \((\cdot)^H\).

### 3.4 CP Based STO Estimation Techniques

For estimating STO, CP and the data part which is the replica of the OFDM symbol will share their resemblances. The two sliding windows having \( W_1 \) and \( W_2 \) can slide to get the similar connection amongst the samples within the windows. The similar connection in between the blocks of CP and the data parts when taken into the sliding windows will take full advantage of getting maximized if CP in an OFDM symbol enters into the beginning of the sliding window. The points which get maximized will help to detect the STO.
If the differences between the CP block and the data parts block is minimized then the similar connection in-between these blocks located in the sliding windows will get maximized. The estimated STO can be obtained by examining the related points so as to sort out by taking the differences between CP blocks and the data part bocks of having $N_G$ samples within the specified sliding windows which is minimized [49]. The mathematical expression can be expressed as

$$\hat{\delta} = \arg \min_{\delta} \left\{ \sum_{i=\delta}^{N_G-1+\delta} |y_t[n + i] - y_t[n + N + i]| \right\}$$

(3.4.1)

If there is the existence of CFO then the performance of the system will be degraded so we approached for the another estimation technique which can take the CFO as the estimating technique which helps in minimizing the differences of the $N_G$ samples of CP in window $W_t$ and the conjugate part in the second window taking its square which can be represented by the equation as [47]

$$\hat{\delta} = \arg \min_{\delta} \left\{ \left( \sum_{i=\delta}^{N_G-1+\delta} |y_t[n + i] - y_t^*[n + N + i]| \right)^2 \right\}$$

(3.4.2)

ML estimation is applied to the end by considering the correlation between the two blocks applied in the two sliding windows. Although correlation maximizes, the performance still degrades which expresses the equation as

$$\hat{\delta} = \arg \max_{\delta} \left\{ \sum_{i=\delta}^{N_G-1+\delta} |y_t[n + i] - y_t^*[n + N + i]| \right\}$$

(3.4.3)

The techniques based on ML still deteriorate the system so we use the different ML technique that increases the log-likelihood function.
\[
\hat{\delta}_{ML} = \arg \min_{\delta} \sum_{i=0}^{N_G-1+\delta} \left[ 2(1-\rho)\text{Re}\{|y_i[n+i]-y_i'[n+N+i]|\} \right. \\
\left. - \rho \sum_{i=\delta}^{N_G-1+\delta} |y_i[n+i]-y_i[n+N+i]| \right]
\]

(3.4.4)

Where, \( \rho = \frac{SNR}{SNR + 1} \) [50]. The another ML estimating techniques that can estimation the STO and CFO at the same period of time which can be expressed with the equation as [51]

\[
\hat{\delta}_{ML} = \arg \max_{\delta} \{ |\gamma[\delta]| - \rho \phi[\delta] \}
\]

(3.4.5)

Now

\[
\gamma[m] = \sum_{n=m}^{m+L-1} y_i[n]y_i'[n+N]
\]

(3.4.6)

Where

\[
\phi[m] = \frac{1}{2} \sum_{n=m}^{m+L-1} (|y_i[n]|^2 + |y_i[n+N]|^2)
\]

(3.4.7)

Here \( L \) specifies the samples used for the windows averaging. \( \gamma[m] \) is the correlation which takes the absolute value part helps in estimating STO which gives the performance of the system robust under the presence of CFO.

### 3.5 Training Symbol Based STO Estimation Techniques

Training symbols can be transferred in the receiver side which can be used for symbol synchronization. The involvement of overhead for the transmission of training symbols which is in contrast with the CP is applied in STO estimation technique where it does not
suffer from the effect of multipath channel. The use of two identical training symbols or a
single OFDM symbol having the repetitive structure is applied. The insertion of zero’s in
between the subcarriers is applied in the pattern which is repetitive in time domain. Once
if the repeated training symbol is being sent through the transmitter side then the receiver
will attempt to find out the CFO by maximizing the similar connections between the two
blocks having the CP and the data part blocks which is being applied with in the sliding
windows. The similar connection between these blocks can be analysed and calculated
through an auto-correlation property with having the training symbol repeated. The
estimation of STO can be estimated using the CP technique which can be applied by
minimizing the differences in-between the two blocks consisting of CP and the data parts
obtaining the samples from the sliding windows. The mathematical equation can be
expressed as [38, 52]

\[
\hat{\delta} = \arg \min_{\delta} \left\{ \frac{1}{N^2} \sum_{i=\delta}^{N-1+\delta} |y_t[n+i] - \hat{y}_t^n[n+\frac{N}{2} + i]|^2 \right\}
\]

(3.5.1)

When maximizing the likelihood function [38]

\[
\hat{\delta} = \arg \min_{\delta} \left\{ \frac{1}{\Sigma_{t=\delta}^{N-1+\delta}} \frac{y_t[n+i]y_t^*[n+\frac{N}{2} + i]}{\Sigma_{t=\delta}^{N-1+\delta} y_t^*[n+\frac{N}{2} + i]^2} \right\}
\]

(3.5.2)

This equation will provide the better accurate output without creating any effect in the
CFO than using of the auto-correlation property.
CHAPTER 4

SIMULATION RESULTS

4.1 RESULT AND ANALYSIS OF TIMING METHODS

4.1.1 Schmidl and Cox

Under ideal conditions, when there is no channel effect and no noise, the timing metric gives a plateau of width equal to the CP. In order to see how the algorithm works, we implemented the algorithm and obtained the timing metric. Figure (6) shows the timing metric under ideal channel conditions. It is obvious that the plateau starts at the sample 135, which is the starting point of the CP, and the plateau runs till the sample 160, which is the start of the symbol. The length of the plateau should reduce down to the CP minus the length of the channel impulse response under the actual conditions.

There are two methods to determine the symbol timing. The first one is just to find the maximum of the metric. The second one is to find the maximum, and the points to the left and right that are 90% of the maximum and then compute the average of these two 90% points to find the symbol timing estimate. The second method is more desirable when the metric has a plateau as shown in Figure (6).

Fig. 6: Timing metric under no noise and distortion condition
4.1.2 Minn and Bhargava

The idea behind this method is to avoid the plateau we found in [39] and shown in figure (7). Another point to note is that the results are based on 1,024 sub-carriers and 10% of the OFDM symbol as CP, which corresponds to 102 samples. This technique is not directly applicable to IEEE 802.11a since there are only 64 sub-carriers and the CP is 16 samples that correspond to one-fourth of the OFDM symbol itself. In the simulations, we did not add the CP. Adding the CP causes the timing metric to have more peaks. Figure (7) shows the timing metric under ideal channel conditions. The middle peak is the peak of interest. The starting point of the training symbol is sample 80, where the timing metric has a peak with the magnitude \( \approx 1 \).

![Timing metric](image)

**Fig. 7:** Timing metric under no noise and distortion condition
4.1.3 Performance of CP based STO estimation

Equation 3.4.2 and 3.4.3 can be used for the time domain estimation techniques which helps in finding the maximum in case of correlation and minimum in case of difference. The vertical line shows the STO. The figure (8) shown below is based on the correlation technique and the mean square difference. The correlation based technique doesn’t provide the satisfactory result when simulating by adding CFO where the mean square difference is not affected when applied CFO as well.
Fig. 8: Performance of CP based STO estimation: maximum Correlation based vs minimum difference based Correlation technique having STO at 4, -4, -3 and 2 with CFO at 0 and 0.5 respectively.

4.2 COMPARISON OF TIMING OFFSET ESTIMATORS

Several methods of timing offset estimation were addressed which focuses on the improvement of timing metric. The training symbol of Schmidl and Cox has two identical halves to estimate the STO. Schmidl method has the timing metric with a large plateau which causes large variance in timing offset estimation. Minn’s and Bhargava proposed a training symbol with more than two identical segments along with flipping the signs of last two segment to obtain a steeper roll-off trajectory at the correct timing position in the timing metric. Minn’s uses the autocorrelation in both halves and then sum the result to
obtain the sharper timing metric. The calculation of correlation to obtain the timing metric in simulation result shows that the variance is quite large. Park’s proposed a different synchronization scheme based on reverse autocorrelation method and achieved a sharper timing metric than Schmidl and Minn. This gives the better performance in slow fading channel and degrades its performance in strong fading in comparison to Schmidl and Cox and the Minn. Timing metric obtained through the parks has two large side lobes which will affect the timing performance. K. Shi and E. Serpedin modified the Minn’s algorithm with the outcome of more advanced timing metric based on maximum likelihood criteria.

The timing metric of different timing estimators is as shown in the figure (9) under no noise and no channel condition. The 512 subcarrier OFDM system with 64 Cyclic Prefix has been considered for the generation of timing metric. The correct timing point is indexed 0 as shown in figure. It is clearly visible from the simulation results that the timing metric of K. Shi and E. Serpedin has better peaks than that of minn not that of park. Park has better peaks but has large side lobes.

![Comparison of timing metrics](image)

Fig. 9: Comparisons of different timing offset estimators based on timing metric
CHAPTER 5

CONCLUDING REMARKS AND FUTURE SCOPE

5.1 CONCLUDING REMARKS

The basic fundamental elements of this thesis were to analyse the necessity for the timing synchronization with its effects on STO estimation. The performance of numerous timing methods were analysed in the literature survey to identify which timing metric gives the better performance. The main motive of this thesis is to analyse and investigate the effects of STO on the OFDM system.

We mainly introduced some of the basic fundamental parameters of OFDM system with its OFDM based standards such as IEEE 802.11 as well as the IEEE 802.16 family. The need for timing synchronization, effects of STO was analysed. Different timing offset estimation methods were analysed with mathematical expressions. Matlab simulations were done on different timing estimation methods based on timing metric and the STO estimation techniques based on training symbol and CP. The result performed under the timing metric shows that which of the methods gives the better performance under the different methods.

Chapter 1 describes the basic introduction of 1G-4G as well as the LTE technology based on its data rate speed, bandwidth and its performance on voice, data, picture etc. it also gives the slight blow on the multiple access techniques that how TDMA, FDMA and CDMA operates, wireless propagation due to which reflection, diffraction and scattering occurs through which fading occurs. Some of the fading channel models were also described.

Chapter 2 includes the OFDM system model description where the mathematical analysis was done on OFDM modulation and demodulation as well as of the cyclic prefix. Application, benefits and the problems associated with the OFDM system were explained to boost up the strength for the next chapter.

Chapter 3 acts as a functional body of the thesis where the synchronization based on timing estimation, need and the importance of timing synchronization, effects of STO were explained but the explanation based on the effects of STO for the different cases and the different timing estimation methods were explained mathematically with simplicity so
that the effects and its better timing metric performance can be clearly identified. Chapter 3 also includes the STO estimation techniques based on CP as well as the training symbol with its effect and its verification is done through mathematical analysis.

Chapter 4 describes the performance of timing metric based on timing offset estimation methods as well as the CP based and the training symbol based STO estimation technique through the simulation results. The comparison based on timing estimation methods were analysed through its simulation results so that these results shows that which of these estimation methods gives the better timing metric. CP based STO estimation technique and the training symbol STO estimation technique were also analysed through simulation results. The result shows that correlation technique doesn’t provide the satisfactory result when simulating by adding CFO where the mean square difference is not affected when applied CFO as well.

5.2 FUTURE SCOPE

Many of the problems have been considered in the OFDM system which has to be estimated properly to enhance the performance of the system so that the researchers are searching the new techniques which always become a new challenge and always waiting for the better techniques. For the further extension of the works by any researchers there is always the gap which will help them to develop the new research work as a future scope. The timing offset estimation whose algorithm can be taken as an extension in channel estimation in the OFDM system as a future scope. The utilization of the proposed method for the timing offset estimation can also be applied as in the MIMO OFDM system. The further implementation of the OFDM system can be applied with a radio frequency technique for the future extension to get the better performance in a real environment. We normally used the IEEE 802.11a standard used in this thesis. There consists of other standards such as IEEE 802.16a as well as IEEE 802.16e which are different from the IEEE 802.11a so that the performance can be analysed based on these standards for the future scope. Timing synchronization methods were analysed in this thesis but the frequency synchronization can also be investigated and analysed by different process through the same training sequence or a different one depending on the technique used for frequency synchronization.
REFERENCES


